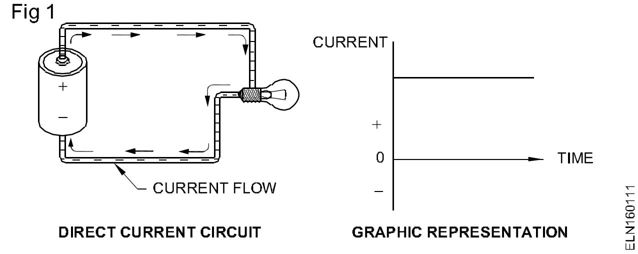
Alternating current - terms & definitions - vector diagrams

Objectives: At the end of this lesson you shall be able to
- **state the features of direct current**
- **list out the advantages of DC over AC**
- **compare the features of DC and AC**
- **explain the generation of alternating current and terms used**
- **state the advantages of AC over DC**

Direct current (DC): Electric current can be defined as the flow of electrons in a circuit. Based on the electron theory, electrons flow from the negative (–) polarity to the positive (+) polarity of a voltage source.

Direct current (DC) is the current that flows only in one direction in a circuit. (Fig 1) The current in this type of circuit is supplied from a DC voltage source. Since the polarity of a DC source remains fixed, the current produced by it flows in one direction only.

Fig 1

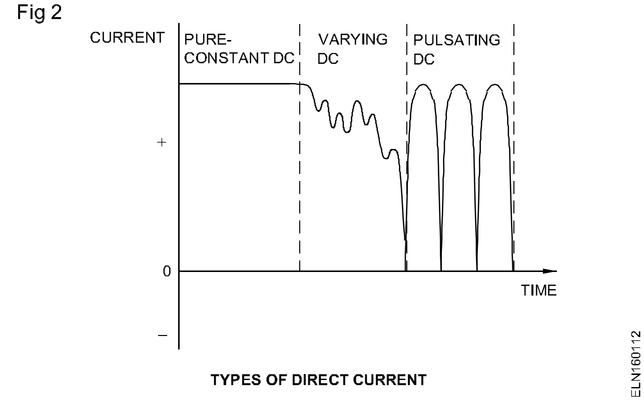


Dry cells are commonly used as a DC voltage source. Both the voltage and polarity of the dry cell are fixed. When connected to a load, the current produced flows in one direction at some steady or constant value.

A direct current flow need not necessarily be constant, but it must travel in the same direction at all times. There are several types of direct current, and all of them depend upon the value of the current in relation to time. (Fig 2)

A constant DC current shows no variation in value over a period of time. Both varying and pulsating DC currents have a changing value when plotted against time. The pulsating DC current variations are uniform, and repeat at regular intervals.

Fig 2



Advantages of DC over AC

1 DC needs only two wires of transmission, while a 3 phase AC may need upto 4 wires.

2 The corona loss associated with DC is negligible while for AC it increases with its frequency.

3 The skin effect is also observed in AC leading to problems in transmission conductor designs.

4 No inductive and capacitive losses.

5 No proximity effect.

Comparison of AC and DC

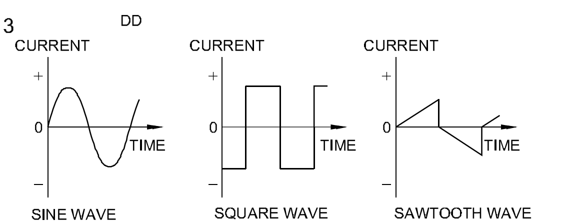
	Alternating current	Direct current
Amount of energy that can be carried	Safe to transfer over longer city distances and can provide more power.	Voltage of DC cannot travel very far until it begins to lose energy.
Cause of the direction of flow of electrons	Rotating magnet along the wire.	Steady magnetism along the wire.
Frequency	The frequency of alternating current is 50Hz or 60Hz depending upon the country.	The frequency of direct current is zero.
Direction	It reverse its direction while flowing in a circuit.	It flows in one direction in the circuit.

	Alternating current	Direct current
Current	It is the current of magnitude varying with time.	It is the current of constant magnitude.
Flow of electrons	Electrons keep switching directions - forward and backward.	Electrons move steadily in one direction or 'forward'.
Obtained from	AC generator and mains.	Cell or battery.
Passive parameters	Impedence.	Resistance only.
Power factor	Lies between 0 to 1.	It is always 1.
Types	Sinusoidal, trapezoidal, triangular, square	Pure and pulsating.

Alternating current (AC): An alternating current (AC) circuit is one in which the direction and amplitude of the current flow change at regular intervals. The current in this type of circuit is supplied from an AC voltage source. The polarity of an AC source changes at regular intervals resulting in a reversal of the circuit current flow.

Alternating current usually changes in both value and direction. The current increases from zero to some maximum value, and then drops back to zero as it flows in one direction. This same pattern is then repeated as it flows in the opposite direction. The wave-form or the exact manner in which the current increases and decreases is determined by the type of AC voltage source used. (Fig 3)

Fig 3



AC WAVE FORMS

Alternating current generation: Alternating current is used wherever a large amount of electrical power is required. Almost all of the electrical energy supplied for domestic and commercial purposes is alternating current.

AC voltage is used because it is much easier and cheaper to generate, and when transmitted over long distances, the power loss is low.

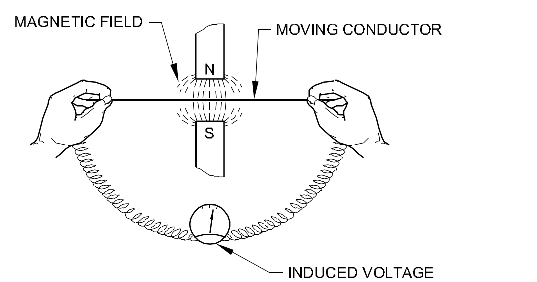
AC equipment is generally more economical to maintain and requires less space per unit of power than the DC equipment.

Alternating current can be generated at higher voltages than DC, with fewer problems of heating and arcing. Some standard values of voltages are 1.1KV, 2.2.KV, 3.3KV for low capacity and 6.6KV (6600V), 11KV(11000V) and 33KV(33000V) for high capacity requirements. The values are increased to 66 000, 110 000, 220 000, 400 000 volts for transmission over long distances. At the load area, the voltage is decreased to working values of 240V and 415V.

The basic method of obtaining AC is by the use of an AC generator. A generator is a machine that uses magnetism to convert mechanical energy into electrical energy. The generator principle, simply stated, is that a voltage is induced in a conductor whenever the conductor is moved through a magnetic field so as to cut the lines of magnetic force.

Fig 4 shows the basic generator principle. A change in a magnetic field around a conductor tends to set electrons in motion. The mere existence of a magnetic field is not enough; there must be some form of change in the field.

Fig 4



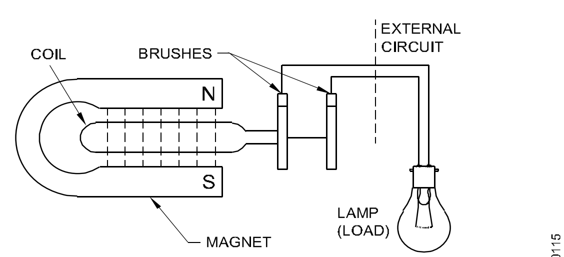
GENERATOR PRINCIPLE

If we move the conductor through the magnetic field, a force is exerted by the magnetic field on each of the free electrons within the conductor. These forces add together and the effect is that voltage is generated or induced into the conductor.

An AC generator produces an AC voltage by causing a loop of wire to turn within a magnetic field. This relative motion between the wire and the magnetic field causes a voltage to be induced between the ends of the wire. This voltage changes in magnitude and polarity as the loop is rotated within the magnetic field. (Fig 5)

The force required to turn the loop can be obtained from various sources. For example, very large AC generators are turned by steam turbines or by the movement of water.

Fig 5



SIMPLIFIED AC GENERATOR

The voltage produced by a single loop generator is too weak to be of much practical value. A practical AC generator has many more turns of wire wound on an armature. The armature is made up of a number of coils wound on an iron core.

The AC voltage induced in the armature coils is connected to a set of slip rings from which the external circuit receives the voltage through a set of brushes. An electromagnet is used to produce a stronger magnetic field.

The sine wave: The shape of the voltage wave-form generated by a coil rotating in a magnetic field is called a sine wave. The generated sine wave voltage varies in both voltage value and polarity.

If the coil is rotated at a constant speed, the number of magnetic lines of force cut per second varies with the position of the coil. When the coil is moving parallel to the magnetic field, it cuts no lines of force.

Therefore, no voltage is generated at this instant. When the coil is moving at right angles to the magnetic field, it cuts the maximum number of lines of force.

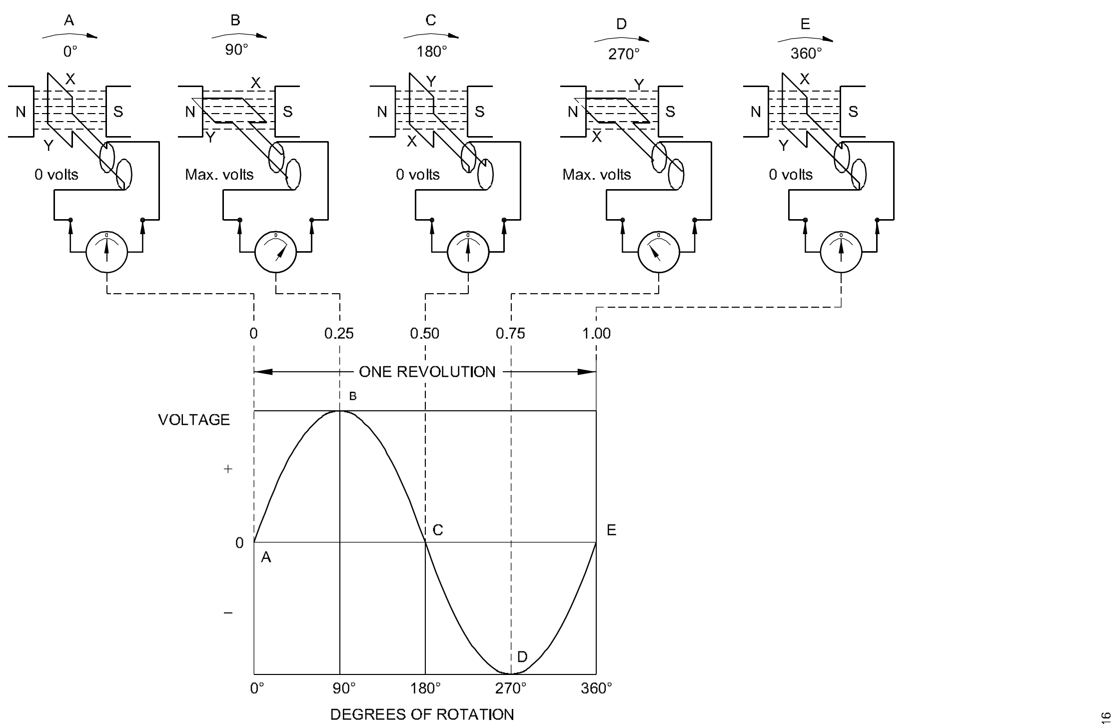
Therefore, maximum or peak voltage is generated at this instant. Between these two points the voltage varies in accordance with the sine of the angle at which the coil cuts the lines of force.

The coil is shown in five specific positions in Fig 6. These are intermediate positions which occur during one complete revolution of the coil position. The graph shows how the voltage increases and decreases in amount during one rotation of the loop.

Note that the direction of the voltage reverses each half-cycle. This is because, for each revolution of the coil, each side must first move down and then up through the field.

The sine wave is the most basic and widely used AC wave-form. The standard AC generator (alternator) produces a voltage of sine wave-form. Some of the important electrical characteristics and terms used when referring to AC sine wave voltage or current are as follows.

Fig 6

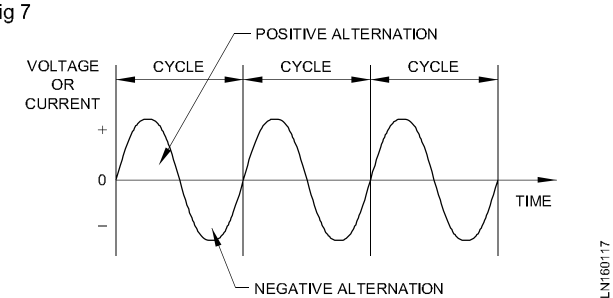


GENERATION OF AN ALTERNATING VOLTAGE: AS THE LOOP ROTATES THROUGH THE MAGNETIC FIELD. THE AMOUNT AND POLARITY OFTHE VOLTAGE CHANGES WITH ANGLE AND DIRECTION OF MOTION.

Cycle: One cycle is one complete wave of alternating voltage or current. During the generation of one cycle of output voltage, there are two changes or alternations in the polarity of the voltage.

These equal but opposite halves of a complete cycle are referred to as alternations. The terms positive and negative are used to distinguish one alternation from the other. (Fig 7)

Fig 7

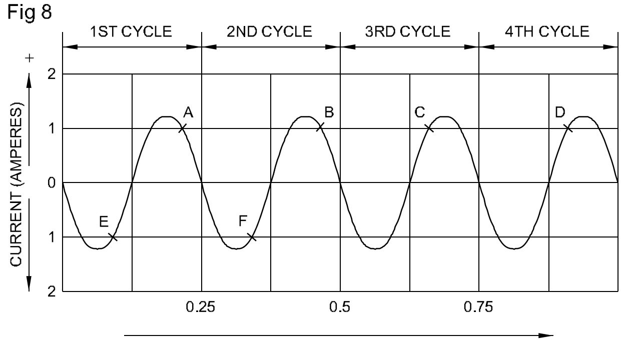


Period: The time required to produce one complete cycle is called the period of the wave-form. In Fig 8, it takes 0.25 seconds to complete one cycle. Therefore, the period (T) of that wave-form is 0.25 seconds.

The period of a sine wave (any symmetrical wave-form) need not necessarily be measured between the zero crossings at the beginning and the end of a cycle. It can be measured from any point in a given cycle to the corressponding point in the next cycle. (See Fig 8-AB, CD or EF.)

Frequency: The frequency of an AC sine wave is the number of cycles produced per second.(Fig 8) The SI unit of frequency is the hertz (Hz). For example, the 240V AC at your home has a frequency of 50 Hz.

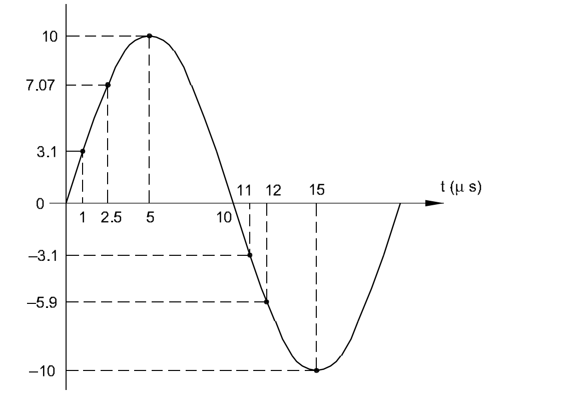
Fig 8



CYCLE, PERIOD, AND FREQUENCY. THE WAVEFORM HAS A PERIOD OF 0.25 SECONDS AND FOUR CYCLES PER SECOND.

Instantaneous value: The value of an alternating quantity at any particular instant is called instantaneous value. The instantaneous values of a sine wave voltage is shown in Fig 9. It is 3.1 volts at 1μs, 7.07 V at 2.5μs, 10V at 5μs, 0V at 10μs, 3.1 volt at 11 μs and so on.

Fig 9

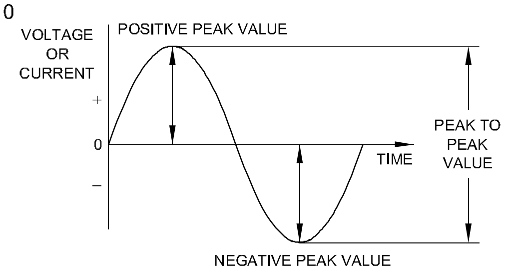


EXAMPLE OF INSANTANEOUS VALUES OF A SINE WAVE VOLTAGE

AC voltage and current values: Since the value of a sine wave of voltage or current continuously changes, one must be specific, while referring to and describing the values of the wave-form . There are several ways of expressing the value of a sine wave.

Peak value or maximum value: Each alternation of the sine wave is made up of a number of instantaneous values. These values are plotted at various heights above and below the horizontal line to form a continuous wave-form. (Fig 10)

Fig 10



PEAK TO PEAK VALUE OF A SINE WAVE

The peak value of a sine wave refers to the maximum voltage or current value. Note that two equal peak values occur during one cycle.

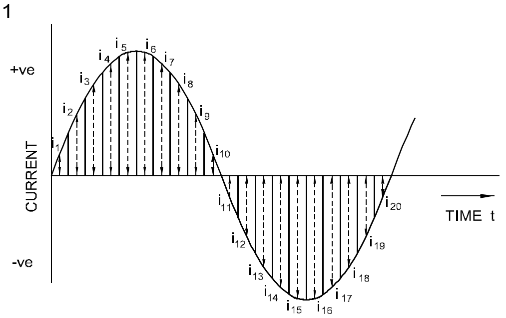
Peak-to-peak value: The peak-to-peak value of a sine wave refers to its total overall value from one peak to the other. (Fig 10) It is equal to two times the peak value.

Effective value: The effective value of an alternating current is that value which will produce the same heating effect as a specific value of a steady direct current. In other words, an alternating current has an effective value of 1 ampere, if it produces heat at the same rate as the heat produced by 1 ampere of direct current, both flowing in the same value of resistance.

Another name for the effective value of an alternating current or voltage is the root mean square (rms) value. This term was derived from a method used to compute the value. The rms is calculated as follows.

The instantaneous values for one cycle are selected for equal periods of time. Each value is squared, and the average of the squares is calculated (values are squared because the heating effect varies as square of the current or voltage). The square root of this is the rms value. (Fig 11)

Fig 11



AVERAGE OF SQUARE OF INSTANTANIOUS VALUES $= \dfrac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_{18}^2 + i_{19}^2 + i_{20}^2}{20}$

By using this method it can be proved that the effective value of a sine wave of current is always equal to 0.707 times its peak value. A simple equation for calculating the effective value of sine wave is:

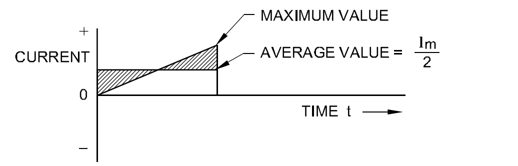
for voltage, $V = 0.707\ V_m$
for current, $I = 0.707\ I_m$

where subscript m refers to the maximum value.

When an alternating current or voltage is specified, it is always the effective value that is meant, unless otherwise stated. Standard AC meters indicate effective values only.

Average value: It is sometimes useful to know the average value for one half cycle. If the current is changed at the same rate over the entire half cycle as in Fig 12, the average value would be one half of the maximum value.

Fig 12



However, because the current does not change at the same rate, another method is used. Find the area covered by the curve over the horizontal axis, then divide that area by the base horizontal length. It has been determined that the average value is equal to 0.637 times the maximum value for sine wave-form i.e.

for voltage, V_{av} = 0.637 V_m

for current, I_{av} = 0.637I_m

where subscript av refers to the average value and subscript m refers to the maximum value.

Form factor (k_f): Form factor is defined as the ratio of effective value to average value of half cycle.

For sinusoidal AC

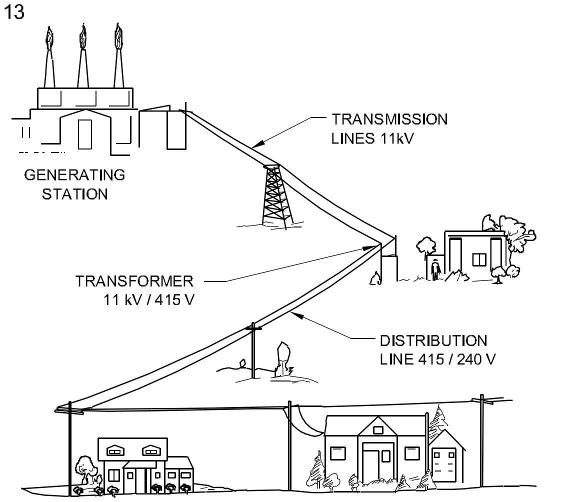
$$k_f = \frac{0.707\ I_m}{0.6637\ I_m} = 1.11$$

where the subscript m refers to the maximum value.

Advantages of AC over DC:

1. AC voltages can be raised or lowered with ease. This makes it ideal for transmission purposes.
2. Large amounts of power can be transmitted at high voltage and low currents with minimum loss.
3. Because the current is low, smaller transmission wires can be used to reduce installation and maintenance costs. (Fig 13)

Fig 13



TRANSMISSION AND DISTRIBUTION LINES FROM GENERATING STATION TO RESIDENCES

DC generators limit their output voltage to 6000V or less. The voltage cannot be raised or lowered through the transformers. Long distance transmission requires heavy cables. AC generators are built with a capacity up to 500000 kilowatts. The DC generators capacity is limited to 10000 kw.

AC motors are less expensive to build, install and maintain than the DC motors. DC motors have one distinct advantage over AC motors: they have better speed control.

- AC is easy to generate than DC.
- It is cheaper to generate AC than DC.
- AC generators take higher efficiency than DC.
- The loss of energy during transmission in negligible for AC in long distance.
- The AC can be easily converted to DC.
- It can easily stepup or stepdown using transformer.
- The value or magnitude of AC can be decreased easily without loss of excess of energy using choke.

Neutral and earth conductors

Objectives: At the end of this lesson you shall be able to
- **describe the purpose of earthing**
- **describe the two types of earthing**
- **differentiate between `neutral' and `earth wire'.**

Earthing: The importance of earthing lies in the fact that it deals with safety. One of the most important, but least understood, considerations in the design of electrical systems is that of earthing (grounding). The word `earthing' comes from the fact that the technique itself involves making a low-resistance connection to the earth or to the ground. The earth can be considered to be a large conductor which is at zero potential.

Purpose of earthing: The purpose of earthing is to provide protection to personnel, equipment and circuits by eliminating the possibility of dangerous or excessive voltage.

There are two distinct considerations in the earthing of an electrical system: earthing of one of the conductors of the wiring system, and earthing of all metal enclosures which contain electrical wires or equipment. The two types of earthing are:

- System earthing
- Equipment earthing.

System earthing: This consists of earthing one of the wires of the electrical system, such as the neutral, to limit the maximum voltage to earth under normal operating conditions.

Equipment earthing: This is a permanent and continuous bonding together (i.e. connecting together) of all non-current carrying metal parts of the electrical equipment to the system earthing electrode.

What is an earthing electrode?: A metal plate, pipe or other conductors electrically connected to the general mass of the earth is known as an earthing electrode. Earth electrodes shall be provided at generating stations, substations and consumer premises (in accordance with the requirements of IS : 3043-1966).

The neutral used in single phase system is to provide return path for load current to the source. Various method of neutral earthing is provided to serve neutral in single phase distribution at substation according to the requirements.

What is an `earth wire'?: A conductor connected to earth and usually situated in proximity to the associated line conductors which is used for equipment earthing is called an earth wire.

The purpose of equipment earthing: By connecting the metal work not intended to carry current to earth, a path is provided for leakage current which can be detected, and, if necessary, interrupted by the following devices.

- Fuses
- Circuit breakers.

Identification: All cores of cables and conductors should be identified at the points of termination, and preferably throughout their length to indicate their function.

Methods of identification may include coloured insulation applied to conductors in manufacture or the application of coloured tape, sleeves or discs (Fig 1). The colours used must be of those specified for the function in Table 52A of IEE wiring regulations.

Fig 1

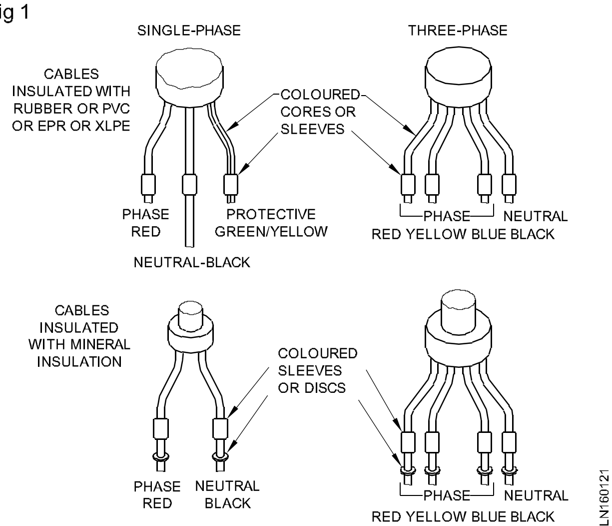


Table 52A of IEE regulations

Colour identification of cores of non-flexible cables and bare conductors for fixed wiring

Function	**Colour identification**
Protective (including earthing)conductor	Green-and-yellow
Phase of ac. single- or three-phase circuit	Red(or yellow or blue*)
Neutral of ac single- or three-phase circuit	Black
Phase R of 3-phase ac circuit	Red
Phase Y of 3-phase ac circuit	Yellow
Phase B of 3-phase ac circuit	Blue
Positive of dc 2-wire circuit	Red
Negative of dc 2-wire circuit	Black
Outer (positive or negative) of dc 2-wire circuit derived from 3-wire system	Red
Positive of 3-wire dc circuit	Red
Middle wire of 3-wire dc circuit	Black
Negative of 3-wire dc circuit	Blue

As alternatives to the use of red, if desired, in large installations, on the supply side of the final distribution board.

Flexible cables and flexible cords: Every core of a flexible cable or flexible cord shall be identifiable throughout its length as appropriate to its function, as indicated in Table 52B of IEE Regulations.

Flexible cables or flexible cords having the following core colours shall not be used; green alone, yellow alone, or any bi-colour other than the colour combination green and yellow. When armoured PVC insulated auxiliary cables or paper insulated cables are used an alternative identification system may be used using numbers, where 1, 2 & 3 signify phase conductors and O the neutral conductor. The number 4 is used to identify any special purpose conductor (Fig 2).

Fig 2

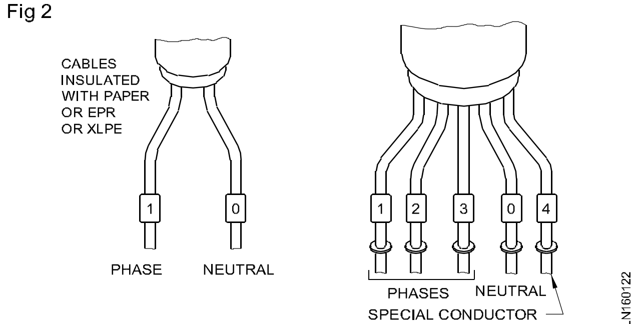


Table 52B of IEE regulations

Colour identification of cores of flexible cables and flexible cords		
No. of cores	**Function of core**	**Colour(s) of core**
1	Phase	Brown
	Neutral	Blue
	Protective (combination)	Green and yellow
2	Phase	Brown
	Neutral	Blue
3	Phase	Brown
	Neutral	Blue
	Protective (combination)	Green and yellow
4 or 5	Phase	Brown or black
	Neutral	Blue
	Protective	Green and yellow (combination)

The colour combination of green and yellow is to be used exclusively for the identification of protective conductors.

Where electrical conduits need to be easily identified from the pipelines of other services such as gas, oil, water, steam, etc. they should be painted orange.

Use of vector diagram

Objectives: At the end of this lesson you shall be able to
- **distinguish between scalar and vector quantity**
- **illustrate the method of drawing vector diagram for two vectors.**

Definition of scalar and vector quantity and phasor

Scalar quantity:A scalar quantity is a quantity which is determined by the magnitude alone, for example energy, volume, temperature etc.

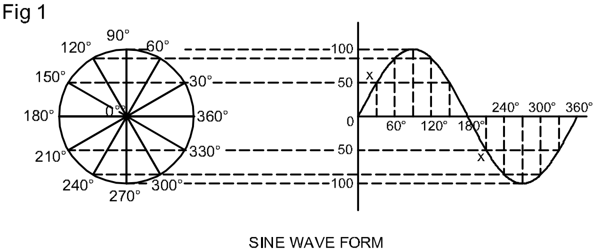
Vector quantity: A vector quantity is a quantity which is represented by straight line with an arrow head to represent the magnitude and direction of it. For example, - force, velocity, weight.

Phasor: Phasor is a vector that is rotating at a constant angular velocity. A straight line with an arrow head is used to represent graphically the magnitude and phase of a sinusoidal alternating quantity (i.e. current, voltage and power) is called phasor.

Plotting a curve of alternating voltage: If the maximum voltage of the alternator is known, the generated voltage can be plotted to form a curve. Draw a circle with the radius representing the maximum value of voltage.

Any convenient scale may be used. Divide the circle into equal parts. (Fig 1) Draw a horizontal line to scale, along which one voltage cycle will be plotted. Divide the line into the same number of equal parts as in the circle. Draw horizontal and vertical lines, as illustrated by the dashed lines in Fig 1. The intersection of the lines represents the value of voltage at that instant. For example, a horizontal and a vertical line intersect at point X.

Fig 1



SINE WAVE FORM

Using the same scale as used for the radius of the circle, the vlaue of voltage can be measured. This value is the emf produced when the coil is cutting the lines of force at a 30-degree angle.

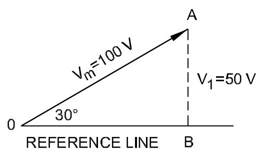
Use of vector diagrams: The change which occurs in the value of an alternating voltage and/or current during a cycle can also be shown by using vector diagrams.

A vector is a line segment that has a define length and direction. A vector diagram is two or more vectors joined together to convey information. Vector diagrams drawn to scale can be used to determine instantaneous values of current and/or voltage.

Scalar quantity	Vector quantity
1. Scalar quantity can be presented by magnitude only, for example - energy, volume etc.	Vector quantity must represent magnitude and direction also, for example - force velocity etc.
2. Addition and substraction of scalar quantities can be done algebraically	Addition and subtraction of vector quantities cannot be done algebracially but by vector summation.

Fig 1 can be analyzed by means of vector diagrams according to the following procedure. Draw a horizontal line as a reference line (Fig 2). Starting at point O and 30 degrees from the reference line, draw OA to scale to represent a maximum voltage (Vm) of 100 volts. From the end of vector OA, draw a vertical dashed line is labelled AB and represents the instantaneous value of voltage (V_i) when the coil is cutting the lines of force at a 30 degree angle. Measure vector AB. It should scale to 50 volts.

Fig 2



VECTOR DIAGRAM DRAWN TO SCALE TO DETERMINE THE VOLTAGE WHEN THE COIL IS MOVING AT A 30° ANGLE TO THE LINES OF FORCE

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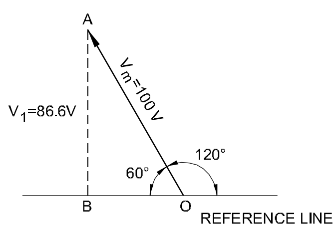
The same procedure can be followed for any degree of rotation. The vector diagram shown in Fig 3 is used to determine the value of voltage when the coil has rotated 120 degrees.

Fig 3



VECTOR DIAGRAM DRAWN TO SCALE. THE COIL HAS ROTATED 120° AND IS CUTTING THE FLUX AT 60°

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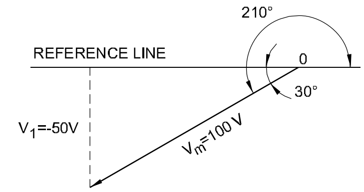
Although the coil has rotated 120 degrees, the angle it is making with the lines of force is only 60 degrees. It is this angle that determines the value of the instantaneous voltage. For example, if the coil rotates 210 degrees, it cuts the lines of force at angle of 30 degrees. (Fig 4)

Fig 4



VECTOR DIAGRAM DRAWIN TO SCALE. THE COIL HAS ROTATED 210° AND IS CUTTING THE FLUX AT 30°. THE EMF HAS REVERSED DIRECTION, INDICATED BY THE -VE SIGN.

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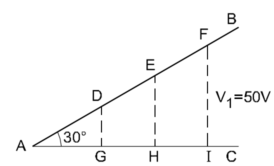
Referring back to Fig 1, it can be seen that each division of the circle can represent vector OA. Vector AB can be represented by points along the voltage curve. The angle between the horizontal diameter of the circle and the radius v_m is the angle at which the coil is cutting the flux. Although vector diagrams are seldom used alone, they are a simple way of presenting a visual illustration of a problem. Vector diagrams are usually used with trigonometric functions.

Many electrical problems are solved through the use of trigonometry. The vector diagrams used with trigonometric functions are generally in the form of triangles and/or parallelograms.(Fig 5 & 6)

Fig 5



VECTOR DIAGRAM FORMING THREE 30° - 60° - 90° TRIANGLES

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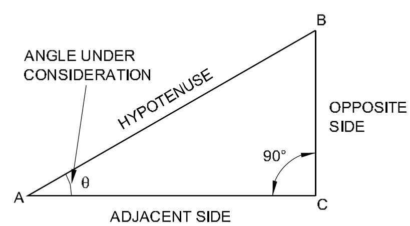
Fig 6



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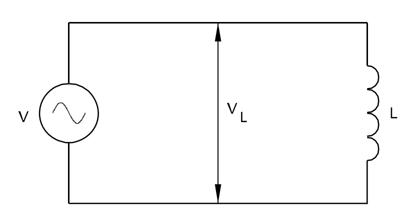
AC simple circuit - with inductance only

Objectives: At the end of this lesson you shall be able to
- **state phase relation between V and I in a pure inductive circuit**
- **state about inductive reactance**
- **state power in pure inductive circuit**
- **define power factor.**

Circuit with pure inductance only: Inductance affects the operation of pure DC circuits only at the instant they are opened and the instant they are closed. In an AC circuit, the current is always changing, and the inductance is always opposing the change. The inductance, therefore, has a constant effect on circuit operation.

A circuit with pure inductance alone can never be formed, because the source, the connecting wires, and the inductor all have some resistance. However, if these resistances are very small and have a much smaller effect on the circuit current than does the inductance, the circuit can be considered as containing only inductance. (Fig 1)

Fig 1



CIRCUIT DIAGRAM

ELN160141

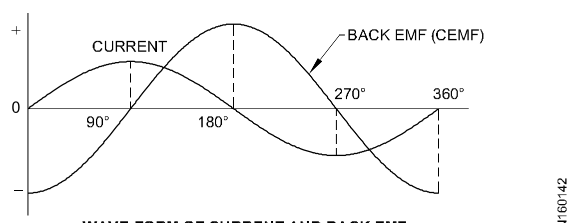
In any AC circuit that contains only inductance, there are three varying quantities. These are (1) the applied voltage, (2) the induced back emf, and (3) the circuit current.

Phase relationship between voltage and current: The phase relationships in an inductance can most easily be understood by considering first the current and the back emf. You know two things about the current and the back emf. One is that the counter emf is maximum when the rate of change of current is the greatest, and is zero when the current is not changing.

The second relationship is that the direction of the cemf is such that it always opposes the current change.

The wave-form in Fig 2 shows one cycle of AC current. The rate of change is greatest where the slope of the wave-form is greatest. You can see that this occurs at those points where the wave-form passes through zero; or at 0,180, and 360 degrees. This means that the highest cemf is generated at 0,180 and 360 degrees, as shown by the wave-forms in Fig 2. Around 90 and 270 degrees, the change is very little; as a matter of fact, at exactly 90 and 270 degrees, where the current change is from rising to falling, the current is momentarily steady. Therefore, the flux lines do not change at those points, and no cemf is induced.

Fig 2



WAVE-FORM OF CURRENT AND BACK EMF

What should be the amount and direction of counter emf (back emf) when the current is passing through zero and in the positive direction?

Since at 0 degrees the current is passing through zero in a positive direction, the cemf must be maximum in the negative direction, in as much as it always opposes the increase in current. Similarly, when the current begins to decrease at 90 degrees, the cemf must be increasing in the positive direction to aid the current flow.

As shown, therefore, the cemf follows Lenz's Law by lagging the current by 90 degrees. You know that the applied voltage is 180 degrees out of phase with the cemf, and so the applied voltage must lead the current by 90 degrees. This is shown in the wave-form in Fig 3. The relationships between the three quantities (current, cemf, and applied voltage) is shown in the wave-forms in Fig 4. We know they are not in phase as in the case of resistive circuits.

Fig 3

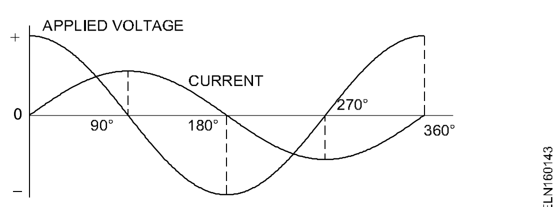
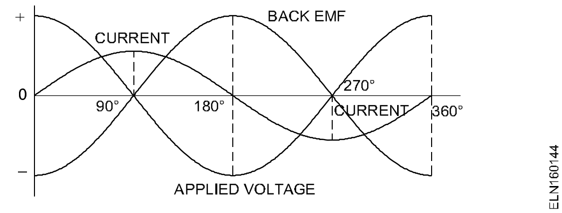


Fig 4



In an inductance: (1) the applied voltage leads the current by 90°; (2) the back emf lags the current by 90° and (3) the applied voltage and the back emf are 180° out of phase.

This is known as 'phase difference'.

Phase difference: If two alternating quantities attain maximum value in the same direction after passing through zero value at different times, they are said to have a phase difference.

Phase difference can be expressed in fractions of a cycle. For more accuracy, phase difference is given in degrees. The terms `lead' and `lag' are used to describe the relative positions in time of two voltages or currents that are not in phase. The one that is ahead in time is said to lead, while the one behind lags. (Figs 5 and 6)

Fig 5

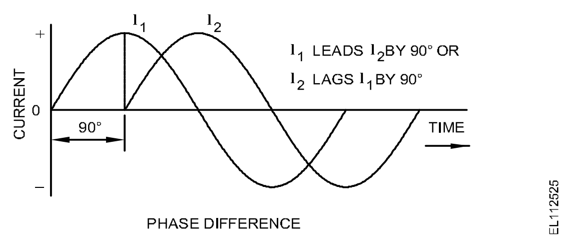
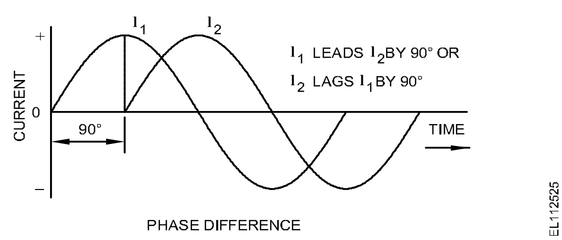


Fig 5



When maximum and minimum points of one voltage or current occur before the corresponding points of another voltage or current, the two are out of phase. When such a phase difference exists, one of the voltages or currents leads, and the other lags.

Phase difference can also be illustrated by a vector diagram. While representing phase difference, the reference quantity is shown on the +ve side of the x axis.(Figs 7 & 8)

Fig 7

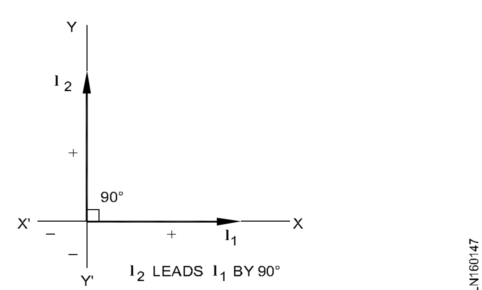
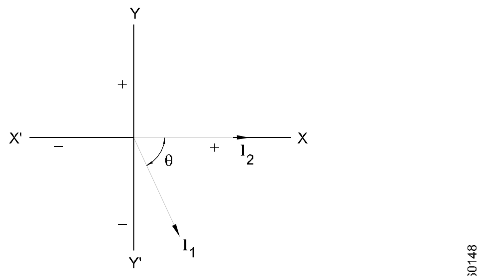


Fig 8



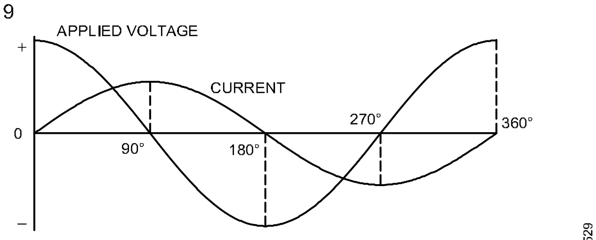
I_1 LAGGING I_2 BY ANGLE θ

The quantity of lead is shown by an angle in an anticlockwise and a lagging quantity is shown by a clockwise angle.

Phase relationship between current and voltage in a circuit with inductance only

When AC voltage is applied to an inductive circuit, the current lags behind the applied voltage by a quarter cycle or by 90°. (Fig 9)

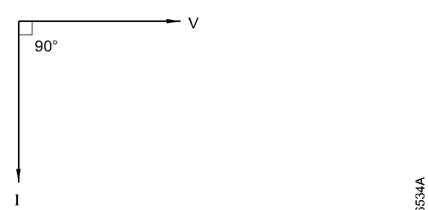
Fig 9



CURRENT LAGS VOLTAGE BY 90°

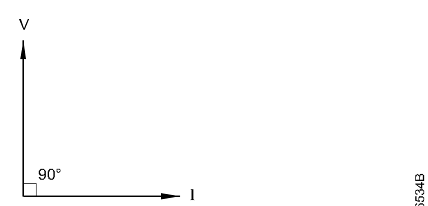
In a purely inductive circuit, the current lags behind the applied voltage by 90°. This is illustrated in the Fig 9 as wave-form. This also can be stated as voltage leads current. The vector diagram for both expressions is given in Figs 10 and 11.

Fig 10



I LAGS BEHIND V BY 90°

Fig 11



V LEADS I BY 90°

Inductive reactance: The cemf acts just like a resistance to limit the current flow. But cemf is discussed in terms of volts, so it cannot be used in Ohm's Law to compute the current. However, the effect of cemf can be given in terms of ohms. This effect is called inductive reactance, and is abbreviated as X_L. Since the cemf generated by an inductor is determined by the inductance (L) of the inductor, and the frequency (f) of the current, the inductive reactance must also depend on these things. The inductive reactance can be calculated by the equation

$$X_L = 2\pi fL$$

where X_L is the inductive reactance in ohms; f is the frequency of the current in cycles per second; and L is the inductance in henrys. The quantity 2π together actually represents the rate of change of the current, usually denoted by the Greek letter `ω' (Omega).

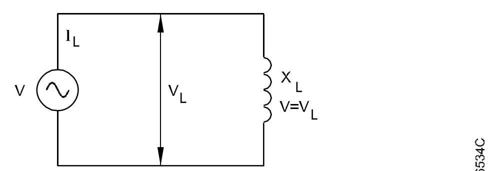
Since 2π = 2(3.14) = 6.28, the Eqn. becomes similarly

$$L = \frac{X_L}{6.28\,f}$$

$$f = \frac{X_L}{6.28\,L}$$

In a circuit containing only inductance, Ohm's Law can be used to find the current and voltage by substituting X_L for R. (Fig 12)

Fig 12



CIRCUIT WITH ONLY X_L

$$I_L = \frac{V_L}{X_L}$$

$$X_L = \frac{V_L}{I_L}$$

$$V_L = I_L X_L$$

where I_L = current through the inductance, in amperes

V_L = voltage across the inductance, in volts

X_L = inductive reactance in ohms

Example: An AC circuit consists of a 20-mH coil operating at a frequency of 1000 kHz. What is the inductive reactance of the coil?

X_L = 6.28fL

= $6.28(1000 \times 10^3)(20 \times 10^{-3})$

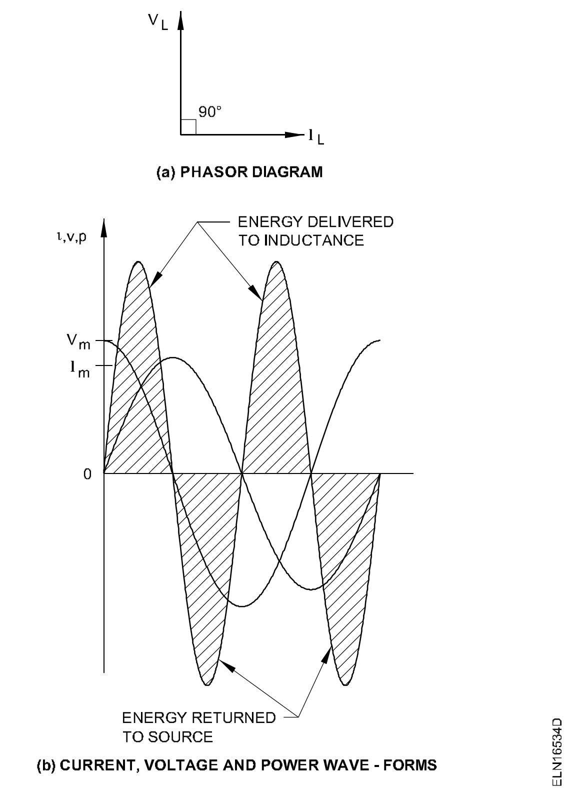
= 12.56×10^4 = 125600 ohms.

Example: What must the inductance of a coil be so that it has a reactance of 628 ohms at a frequency of 40KHz?

$$L = \frac{X_L}{6.28f} = \frac{628}{6.28(40 \times 10^3)} = 2.5 \times 10^{-3} = 2.5\text{ mH}$$

Power in pure inductance: If an AC circuit contains only inductance, the voltage and current are 90° out of phase, as shown by the phasor and wave diagrams in Fig 13. The result of multiplying the V and I wave-forms is a power curve that again has a frequency twice that of the source is as shown in Fig 13. However, over a complete cycle of input voltage, the power curve has an average value of zero. That is, the power curve shows an equal alternation of positive and negative power above and below the zero time axis.

Fig 13



Positive and negative power: (Fig 13) The shaded portion of the power curve above the zero axis represents energy being delivered to the inductor (or load) from the source. This positive power actually represents a storage of energy in the magnetic field of the inductance.

The shaded portion of the power curve below the zero axis represents energy returned to the source from the inductor. This negative power indicates that a flow of energy is taking place in the opposite direction(from load to source) when the coil's magnetic field collapses.

The average true power, P, is zero, in a pure inductance.

In AC circuits,

Power = VI Cos ϕ watts

where ϕ is the phase angle between voltage and current.

As the phase angle between V & I in pure inductive circuit is 90°, Cos 90° is zero.

Therefore P = V x I x (zero) = zero.

The term Cos ϕ is known as `power factor'.

Reactive power: However, the source must be capable of delivering power for a quarter of a cycle, even though this power will be returned during the next quartercycle. This stored or transferred power is called reactive power, $P_{q.}$

In the case of a purely inductive circuit, the reactive power is given by

$P_q = V_L I_L$ volt-amperes reactive (var)

where P_q is the reactive power in volt-amperes reactive, var

V_L is the voltage across the inductance in volts

I_L is the current through the inductance in ampere.

Since $V_L = I_L X_L$

then $P_q = I_L^2 X_L$ var.

where X_L is the inductive reactance in ohms. Note how the equations for relative power are similar to those for true power with X_L used in place of R. But we must remember to use var for the unit of reactive power, not watts.

Example: Calculate the reactive power of a circuit that has an inductance of 4 H when it draws 1.4 amps from a 50Hz supply.

Solution

$X_L = 2\pi fL = 2\pi \times 50Hz \times 4H = 1256$ ohms

$P_q = I_L^2 X_L = (1.4A)^2 \times 1256$ ohms = 2462 vars

= 2.462 kvar

Note that 1 kvar = 1 kilo-var = 1000 vars.

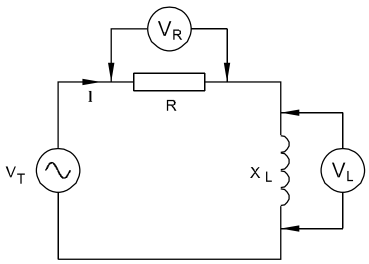
A.C. circuit with R & L in series

Objectives: At the end of this lesson you shall be able to
- **state the voltage and current relationship**
- **determine impedance of a series circuit with RL in series**
- **calculate power in a series circuit (with RL in series)**
- **calculate the power factor in RL series circuit.**

When resistance and inductance are connected in series, or in the case of a coil with resistance, the rms current I_L is limited by both X_L, and R however the current I is the same in X_L and R since they are in series, the voltage drop across R is V_R = IR and the voltage drop across X_L is $V_L = IX_L$. The current I through X_L must lag V_L by 90° because this is the phase angle between current through an inductance and its self-induced voltage. The current I through R,and its IR voltage drop,are in phase and so the phase angle is 0°.

Now let us apply the principle of phasor representation to a series circuit containing pure resistance and pure inductance. (Fig 1)

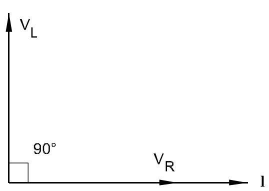
Fig 1



CIRCUIT WITH R & L IN SERIES

Since we are considering a series circuit, it is convenient if we draw the current phasor in the horizontal reference position because it is `common' to both the resistor and inductor. Superimposed upon this phasor is the voltage phasor across the resistor V_R. This is because the current and voltage are always in phase with each other in a pure resistor. (Fig 2)

Fig 2



VECTOR DIAGRAM OF R & L SERIES CIRCUIT

Similarly, the voltage phasor across the inductor V_L is drawn 90° ahead of the current I in other words leading the current phasor. This is because, as we know, the current always lags the inductor voltage by 90° in a pure inductance.

However, these two voltages are 90° out of phase with each other. This means that the total voltage across the series combination cannot be obtained simply by adding V_R to V_L algebraically. We must take into account the angle between them.

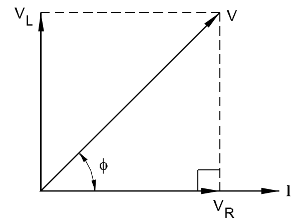
The applied voltage V is the (phasor) sum of V_R and V_L with the phase angle added.

This phasor addition can be carried out simply by constructing a parallelogram (a square in this case) and drawing the diagonal. This is shown in Fig 3. Clearly, the phasor sum V is less than the algebraic sum of V_L and V_R. Also, because V is the hypotenuse of a right-angled triangle, V is given by

$$V^2 = V_R^2 + V_L^2$$

Impedance of a series RL circuit: The total opposition to current in a series, RL circuit, is called the impedance Z. It is the ratio of the total applied voltage V to the current I. Impedance is measured in ohms as are resistance and inductive reactance. But, as shown by the following, impedance is the vector sum of resistance and reactance.

Fig 3

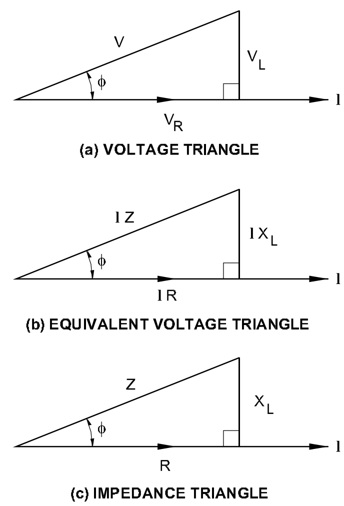


PHASOR DIAGRAM OF R & L SERIES CIRCUIT

Consider the `voltage triangle' for a series, RL circuit, as shown in Fig 4a. This is similar to the phasor diagram in Fig 3 with V_L transferred to make a closed triangle.

Given $V^2 = V_R^2 + V_L^2$ and $V_R = IR$ and $V_L = IX_L$

Fig 4



then $V = \sqrt{(IR)^2 + (IX_L)^2}$

$$= \sqrt{I^2R^2 + (I^2X_L)^2}$$

$$= \sqrt{I^2(R^2 + X_L^2)}$$

$$= I\sqrt{R^2 + X_L^2} \text{ and } \frac{V}{I} = \sqrt{R^2 + X_L^2}$$

But $\frac{V}{I}$ is the impedance Z.

Therefore, $Z = \sqrt{R^2 + X_L^2}$ ohms

where Z is the impedance in ohms

R is the resistance in ohms

X_L is the inductive reactance in ohms

and $I = \frac{V}{Z}$ amperes (A).

We can also see from Fig 4b & 4c that, if the impedance and phase angle are known, we can obtain the resistance and inductive reactance.

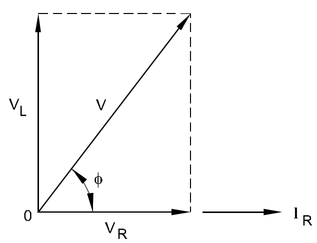
$R = Z \cos \phi$

$X_L = Z \sin \phi$

where ϕ is the angle between Z and R.

Power in a series RL circuit: We have seen that inductance is always accompanied by resistance. Thus coils in motors, generators, relay coils etc. contain both resistance and inductance. When an AC voltage is applied, the current I is neither in phase nor 90° out of phase with the applied voltage V as shown in Fig 5.

Fig 5



CURRENT REFERENCE VECTOR

This means, unlike pure resistance and pure reactance, the product of the voltmeter and ammeter readings in Fig 6 is a combination of the true and (quadrature) reactive power. We call the product of total V and total I apparent power Ps. Since it is neither true power in watts nor reactive power in vars, we use a new unit - the volt ampere, VA to measure the apparent power.

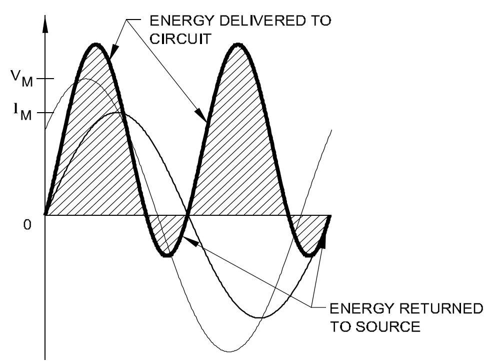
P = V x I volt-amperes (VA)

where P is the apparent power in volt amperes VA,

V is the total applied voltage in volts V,

I is the total circuit current in amperes A.

Fig 6



CURRENT, VOLTAGE, AND POWER WAVE - FORMS

Power triangle: In AC circuit we had identified three types of power.

- True power in watts as in circuit with resistors only.
- Reactive power in vars as in the case of pure inductive or pure capacitive circuit.
- Apparent power in VA as in the case of circuits with R and L or R & C. All the three are interrelated.

We know in a series RL circuit

$$V = \sqrt{V_R^{\ 2} + V_L^{\ 2}}$$

Therefore $V \times I = \sqrt{(V_R \times I)^2 + (V_L \times I)^2}$

But V x I = apparent power in VA

V_R x I = true power in watts

V_L x I = reactive power in vars

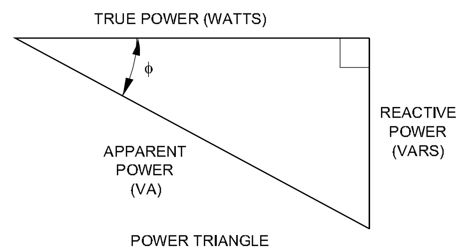
Therefore,

(apparent power)^2 = (true power)^2 + (reactive power)^2

or $$VA = \sqrt{(W^2) + (VAR^2)}$$

This relation can be represented in a power triangle, as in Fig 7.

Fig 7



POWER TRIANGLE RELATIONS FOR A SERIES RL CIRCUIT

Fig 7 shows the apparent power as represented by the hypotenuse of the right angled triangle. The true power is the product of the current and voltage in phase with each other, and is drawn horizontally. The out-of-phase product of V_L and I gives the reactive power, and is drawn vertically downward. This is a convention used to show a lagging, inductive, reactive power corresponding to a lagging current. (A capacitive reactive power is drawn vertically upward, corresponding to a leading current.)

We can also have other relations.

$W = VA \cos \phi$

$VAR = VA \sin \phi$

Power factor: The ratio of the true power delivered to an AC circuit compared to the apparent power that the source must supply is called the power factor of the load.

If we examine any power triangle, as in Fig 7, we see that the ratio of the true power to the apparent power is cosine of the angle Ø.

$$\text{Power factor} = \frac{W}{VA} = \cos \phi$$

As $W = V_R \times I$ and

$VA = V \times I$ also

$V_R = I \times R$

$= I \times Z$

power factor must also be equal to $\frac{V_R}{V}$ and to $\frac{R}{Z}$

Power factor (PF) $= \frac{W}{VA} = \frac{V_R}{V} = \frac{R}{Z} \cos\phi$

What should be the power factor for a circuit containing pure resistance only?. As the phase angle Ø between current and voltages is $\phi = 0$.

$\cos\phi = 1$ and PF = 1.

Similarly the power factor for circuit containing pure inductance or pure capacitance only is zero as

$\cos\phi = \cos 90° =$ zero.

Example: An inductive coil with a resistance of 10 ohms and inductance of 0.05 henry is connected across 240 volt 50 cycle AC mains.

Calculate

i) current taken by the coil

ii) power factor of the circuit

iii) power consumed, and answer

iv) whether the current in the circuit is lagging or leading.

Solution

$X_L = 2\pi fL = 2 \times 3.142 \times 50 \times 0.05 = 15.7$ ohms

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (15.7)^2}$$

$$= \sqrt{346.49} = 18.6 \text{ ohms}$$

i I = (240/18.6) = 12.9 Amps

ii Power factor $= \frac{R}{Z} = \frac{10}{18.6} = 0.537$

iii Power consumed $= I^2R = (12.9)^2 \times 10$

$= 166.72 \times 10$

$= 1667$ W.

iv The current is lagging in the circuit.

Example: An inductive circuit has a resistance of 2 ohms in series with an inductance of 0.015 henry. Find (i) current and (ii) power factor when connected across 200 volt 50 cycles per second supply mains.

Solution

$X_L = 2\pi fL = 2 \times 3.142 \times 50 \times 0.015 = 4.71$ ohms

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(2)^2 + (4.71)^2}$$

$$= \sqrt{4 + 17.39} = \sqrt{26.19}$$

i. $I = \frac{200}{5.11} = 39.13$ amps

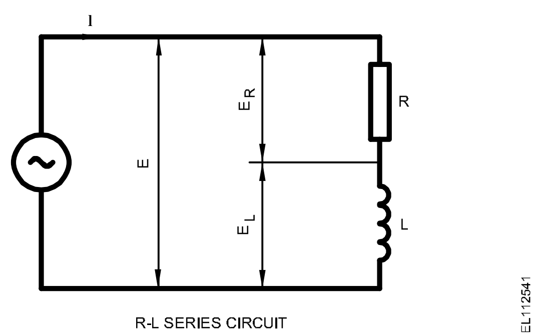
ii. Power factor $= \frac{R}{Z} = \frac{2}{5.11} = 0.39$

Phase relation between V & I in R - L series circuit

Objectives: At the end of this lesson you shall be able to
- **explain how to add and subtract vector quantities**
- **represent voltage of R-L circuit by vector .**

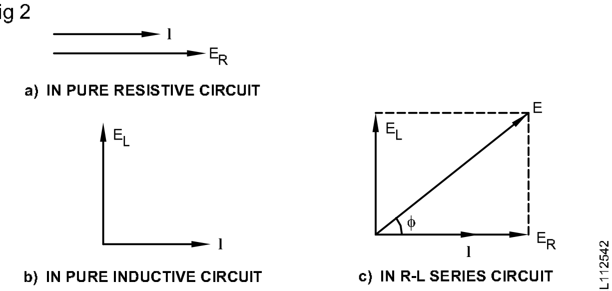
Consider circuit diagram as shown in Fig 1.

Fig 1



R-L SERIES CIRCUIT

Fig 2



When an alternating voltage is applied to a pure inductor, the resultant alternating current through the inductor is lagging by 90° with the supply voltage due to presence of counter emf. (Fig 2b). As current I is shown first Fig 2(a) the voltage across resistor 'R' i.e. this in phase with current. (Fig 2a & 3b)

The voltage across inductor (L) E_L is 90° leading with current I. (Figs 2b and 3c). Hence the applied voltage E is obviously the resultant of E_R & E_L. It is obtained by simply summing up the instantaneous values of E_R and E_L. (Figs 2c & 3d)

Addition and subtraction of vectors by parallelogram method

Addition of two vectors: Two vectors OA & OB are acting on the same point '0' at an angle a as shown in

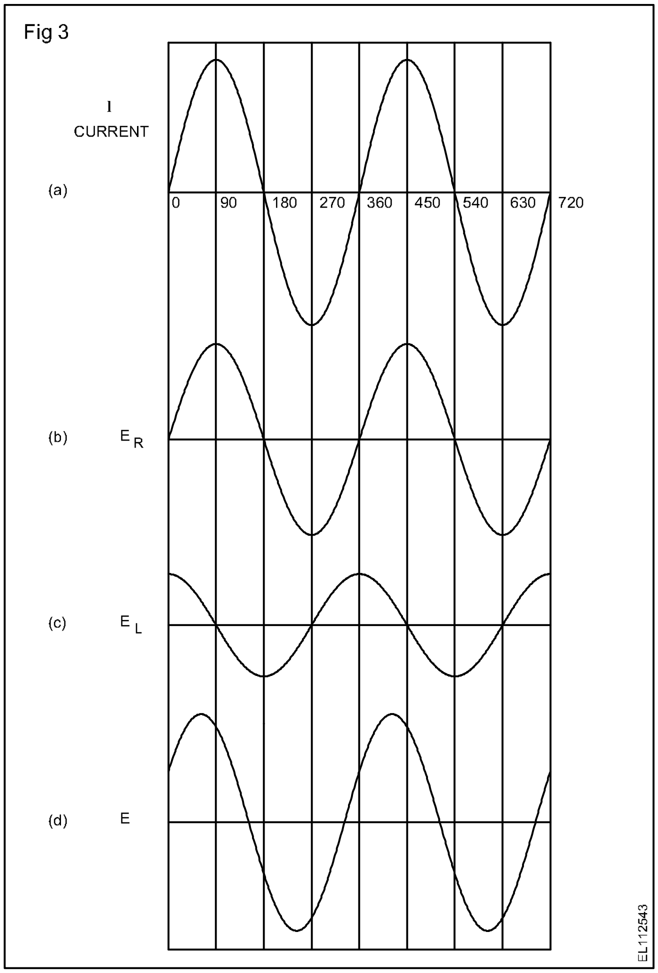
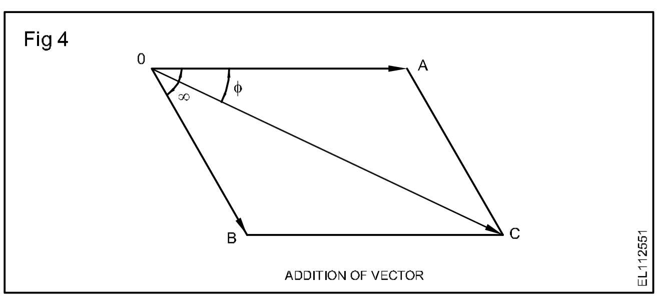


Fig 4. Both vectors can be added by the parallelogram method. On completion of parallelogram OACB, draw the diagonal 'OC' from point 0.

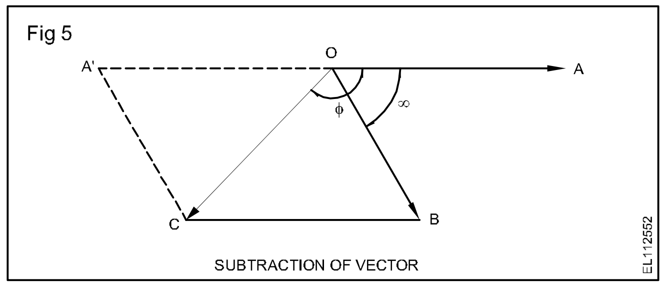


Now 'OC' represents the resultant vector of both vectors.

Subtraction of two vectors: If vector OA is to be subtracted from vector OB (as shown in Fig 5) $(\overline{OB} - \overline{OA})$ then OA is produced backward so that OA' = OA complete parallelogram OBCA'. The diagonal OC drawn from point '0'' of the parallelogram represents the resultant of OA & OB.

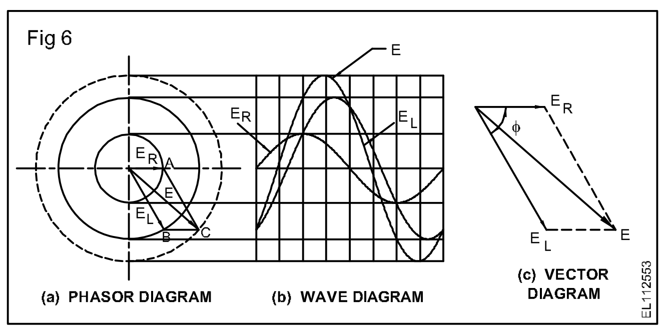
$$\overline{OC} = (\overline{OB} - \overline{OA})$$

Addition of voltage across resistance and inductance connected in series by vector method to compare with supply voltage.



Phasor is representation of sinusoidal quantities. Hence two electrical quantities can be represented by a phase diagram as shown in Fig 6a and wave form a shown by Fig 6b.

Two electrical quantities both (E_R & E_L) voltage can be added by the vector diagram methods as shown in Fig 6c.

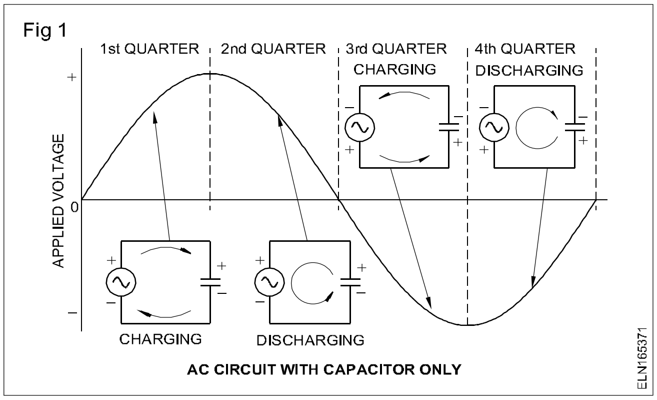


AC Simple circuit - with capacitor only

Objectives: At the end of this lesson you shall be able to
- **explain AC circuit with capacitor only**
- **state phase relation between V and I**
- **state power in pure capacitance only.**

Circuit with capacitance only: In an AC circuit, the applied voltage as well as the current it produces, periodically changes direction. (Fig 1) A capacitor in an AC circuit is first charged by the voltage being applied in one direction. Then, when the applied voltage starts to decrease, less current flows, but the capacitor is still being charged in the same direction. As a result, as the applied voltage continues to drop, the voltage developed across the capacitor becomes greater.

Fig 1



AC CIRCUIT WITH CAPACITOR ONLY

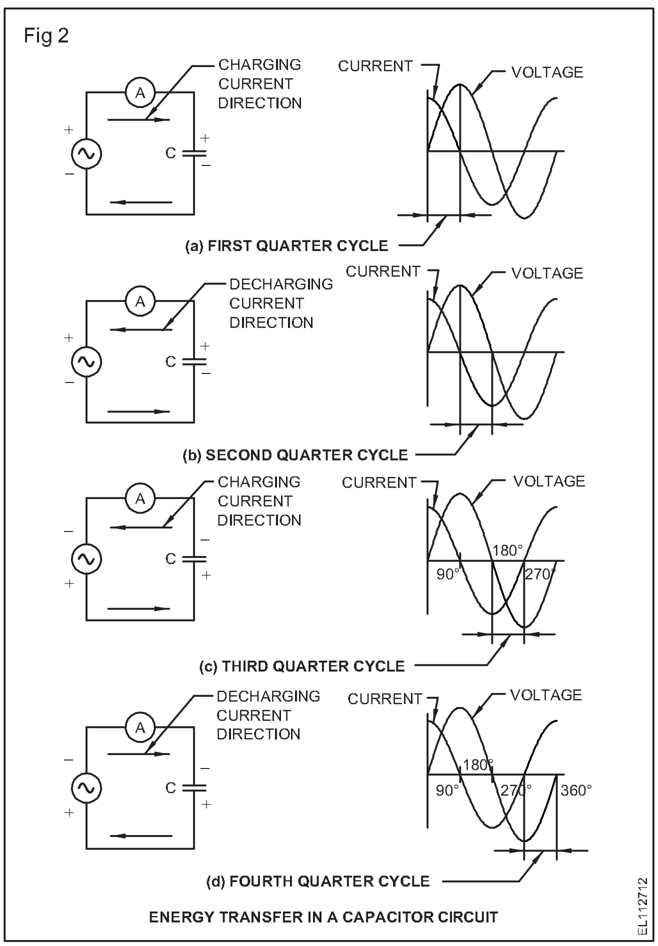
The capacitor then acts as the source, and starts discharging. The capacitor becomes fully discharged when the applied voltage drops to zero and reverses its direction. Then the capacitor starts charging again, but in the same direction in which it was previously discharging.

This continues until the applied voltage again starts to drop, and the events repeat themselves. This alternate charging and discharging, first in one direction, and then in the other, occur during every cycle of the applied AC. An AC current, therefore, flows in the circuit continuously.

It can be said, then, that although a capacitor blocks DC it passes AC.

Voltage and current realtionship: When an AC voltage source is connected across a capacitor, maximum current flows in the circuit the instant the source voltage begins its sinusoidal rise from zero. (Fig 2a)

Fig 2



ENERGY TRANSFER IN A CAPACITOR CIRCUIT

This is because the plates are in neutral state and present no opposing electrostatic forces to the source terminals. Therefore, as you can see by Ohm's Law, if the opposition to the current flow is very, very low, a small applied voltage can cause considerable current to flow.

As the source voltage rises, however, the charges on the capacitor plates, (which result from the current flow) build up. The charge voltage, then, presents an increasing opposition to the source voltage and so the current decreases.

When the source voltage reaches its peak value, the charge voltage across the capacitor plates is maximum. This charge is sufficient to completely cancel the source voltage, so that the current flow in the circuit stops.

As the source voltage begins to decrease, the electrostatic charge on the capacitor plates becomes greater than the potential of the source terminals, and so the capacitor starts to discharge. (Fig 2b)

Is the current flow in the same direction when the voltage decreases?

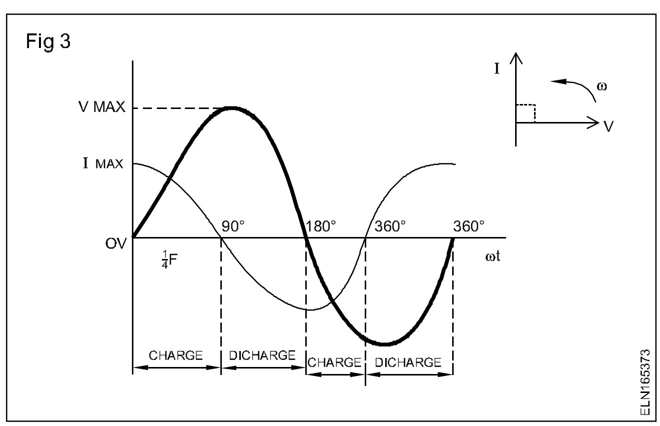
The direction of electron flow is opposite to the direction taken by the electrons during capacitor charging. Thus, at the point that the applied voltage passes through its maximum value and begins decreasing, the current in the circuit passes through zero and changes direction. As can be seen from the graph, this constitutes a 90-degree phase difference, with the current leading the applied voltage.

This 90-degree difference is maintained throughout the complete cycle of applied voltage. When the applied voltage has dropped to zero, the circuit current has increased to its maximum in the opposite direction, and when the voltage reverses the direction, the current begins decreasing. (Fig 2c) Therefore, the voltage applied to a capacitor is said to lag the current through the capacitor by 90 degrees. Or, the current through a capacitor leads the applied voltage by 90 degrees.

Hence the current increases from zero to maximum when voltage starts decreasing from the peak value in the opposite direction to zero. (Fig 2d)

The phase relation between voltage and current in a pure capacitor circuit is shown in the wave-form and vector diagram. (Fig 3)

Fig 3



Capacitive reactance: The opposition offered to the flow of current by a capacitor is called capacitive reactance, and is abbreviated Xc. Capacitive reactance can be calculated by:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{c}$$

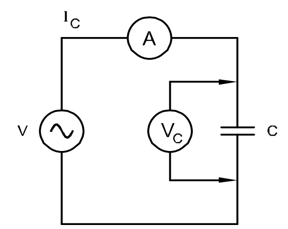
where 2π is approximately 6.28

f is the frequency in Hz

C is the capacitance in farad and $\omega = 2.\pi.f$.

Like its inductive counterpart - inductive reactance, capacitive reactance is expressed in ohms. Ohm's Law can also be applied to a circuit containing capacitive reactance only. (Fig 4)

Fig 4



PURE CAPACITIVE CIRCUIT

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$$V_C = I_c X_c$$

$$I_C = \frac{V_C}{X_C}, \qquad X_C = \frac{V_C}{I_C}$$

where, I_c is current through capacitor in amps

V_C is the voltage across the capacitor in volts

X_C is the capacitive reactance in ohms.

Example: A 10 micro-farad capacitor is connected to a 200V 50Hz supply. Find the current taken.

Solution

$$I_C = \frac{V_C}{X_C}, \qquad V_C = V$$

where X_c is the capacitive reactance.

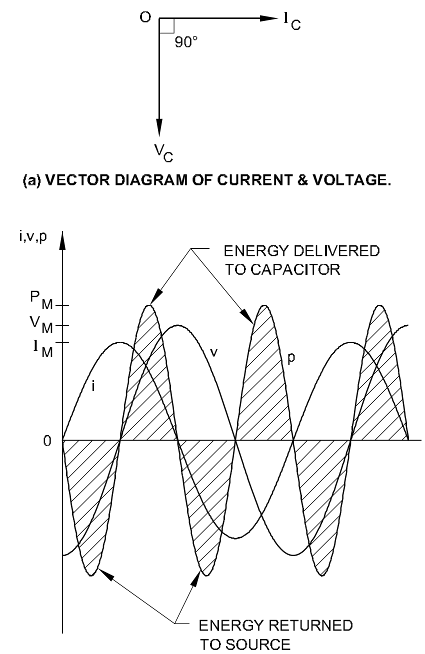
$$X_C = \frac{1}{2\pi fc} = \frac{10^6}{2\pi \times 50 \times 10}$$

$$X_C = \frac{1000}{\pi} = 318.4 \text{ ohms}$$

$$I_C = \frac{200}{318.4} = 0.628 \text{ amps}$$

Power in pure capacitance: For pure capacitance, the voltage and current are 90° out of phase with each other, the current leading as shown by the phase diagram in Fig 5a.

Fig 5



(b) CURRENT, VOLTAGE AND POWER FORMS.

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The product of v and i gives a power curve as shown in Fig 5b. We see that the energy delivered to the capacitor and stored in the electric field is represented as a positive quantity. A quarter of a cycle later, all of this energy is returned to the source, as the capacitor discharges. Thus the average true power, P, is zero in a pure capacitance.

However, reactive power P_q is drawn by the capacitor, and the source must be able to supply this power.

For a purely capacitive circuit, the reactive power is given by

$P_q = V_c I_c$ volt-amperes reactive (var)

where

P_q is the reactive power in volt-amperes reactive, var

V_c is the voltage across the capacitance in volts

I_c is the current through the capacitance in amperes.

Since $V_c = I_c X_c$

then $P_q = I_c^2 X_c$ var

and $P_q = \frac{V_C^2}{X_C}$

where X_c is the capacitive reactance in ohms.

Again the equations for reactive and true power are similar with X_c used in place of R. We must use vars, not watts, for the reactive power.

As in the case of pure inductive circuit, the power factor of the pure capacitive circuit is also zero.

Why is it so?

This is because the angle between the current and voltage in a capacitive circuit is 90°. Result cos ϕ = 0.

Example: A reactive power of 100 vars is drawn by a 10 micro farad capacitor due to a current of 0.87A. Calculate the frequency.

Solution

$$X_C = \frac{P_q}{I_C^2} = \frac{100 \text{ vars}}{(0.87)^2} = 132 \text{ ohms}$$

Therefore, $f = \frac{1}{2.\pi.X_C.C}$

$$= \frac{1}{2\pi \times 132 \text{ ohms} \times 10 \times 10^{-6} \text{F}}$$

= 120 Hz.

Power and power factor in AC single phase circuit

Objective: At the end of this lesson you shall be able to

- **calculate power and power factor of a single phase AC circuit from the given relevant values.**

Power in pure resistance circuit: Power can be calculated by using the following formulae.

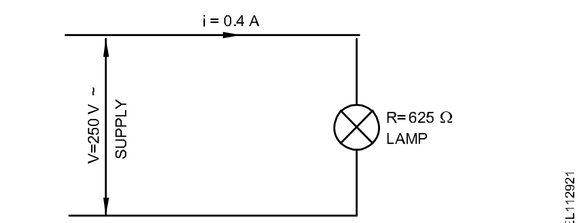
1) $P = V_R \times I_R$ watts

2) $P = I^2_R\ R$ watts

3) $P = \frac{E^2}{R}$ watts

Example 1: Calculate the power taken by an incandescent lamp rated 250V when it carries a current of 0.4A if the resistance is 625 ohms.(Fig 1)

Fig 1



$P = V_R \times I_R$

= 250 x 0.4

= 100 watts.

Alternately

$P = I^2R$

= 0.4 x 0.4 x 625

= 100 watts

$$\text{or } P = \frac{E^2}{R} = \frac{250^2}{625}$$

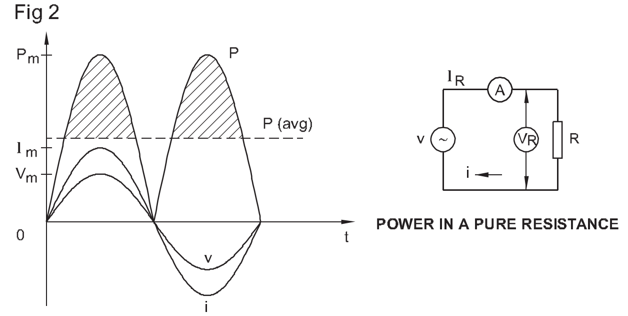
$$P = \frac{250 \times 250}{625}$$

= 100 watts.

Since the current and voltage are in phase, the phase angle is zero and the power factor is unity. Therefore, the power can be calculated with voltage and current itself.

Example 2: A wattmeter connected in an AC circuit indicates 50W. The ammeter connected in series with the load reads 1.5A. Determine the resistance of the load.

Fig 2



CURRENT, VOLTAGE, AND POWER WAVEFORMS

Solution

Known: $P = I_R^2 R$

The circuit arrangement and wave-forms of I, V and P are shown in Fig 2.

Given: I = 1.5 amperes

P = 50 watts.

Therefore,

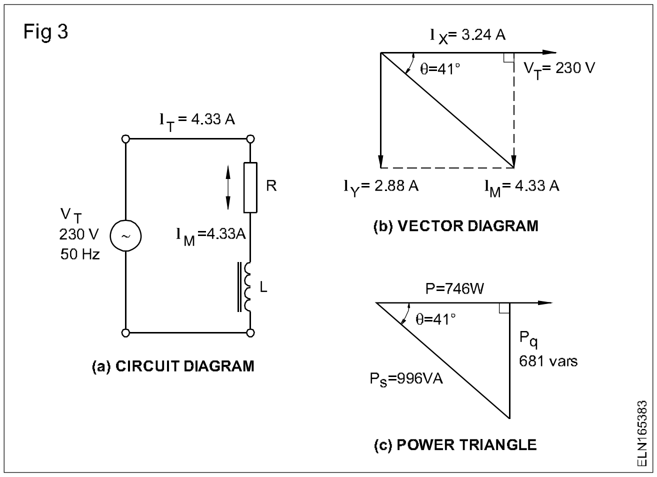
$$R = \frac{P}{I^2 R} = \frac{50W}{(1.5)^2} = 22.2 \text{ ohms}$$

Power in pure inductance: If an AC circuit contains only inductance, the voltage and current are 90° out of phase, and the circuit of the instantaneous values of voltage and current gives with positive and negative power. Net result is the power consumed in a pure inductive circuit is zero.

Power in pure capacitance: If an AC circuit contains only capacitor, the voltage and current are 90°. Out of phase and the product of instantaneous values of voltage and current gives both positive and negative power. Net result is the power consumed in a pure capacitive circuit is zero.

Most industrial installations have a lagging PF because of the large number of AC induction motors that are inherently inductive.

Effect of a low power factor: To show the important effect of the power factor, let us consider a 240V, 50 Hz, 1 hp motor. Let us assume that it is 100% efficient so that it draws a true power of 746 W. Such a motor has a typical power factor of 0.75 lagging. (Fig 3)



To deliver 746 W from 240V at a power factor of 0.75 requires a current of

$$I = \frac{P}{V \times Cos} A$$

$$= \frac{746W}{240V \times 0.75} = 4.144\ A$$

Now let us assume that we can modify the motor in some way to make the power factor unity (I). The current now required is

$$I = \frac{P}{V \times Cos}$$

$$I = \frac{746W}{240V \times 1} = 3.108A$$

Evidently, it requires a higher current to deliver a given quantity of true power if the power factor of the load is less than unity. This higher current means that more energy is wasted in the feeder wires serving the motor. In fact, if an industrial installation has a power factor less than 85% (0.85) overall, a `power factor penalty' is assessed by the electric utility company. It is for this reason that power factor correction is necessary in large installations.

Power factor correction: In order to make the most efficient use of the current delivered to a load we desire a high PF or a PF that approaches unity.

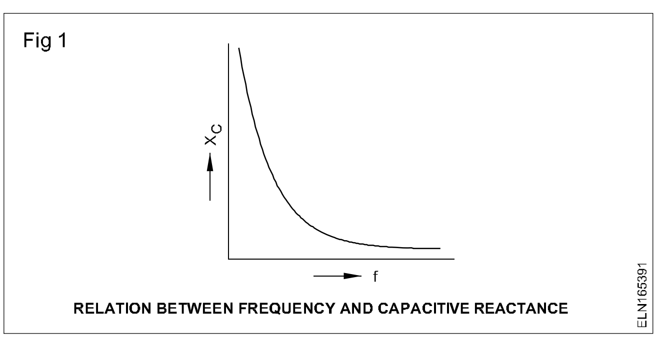
A low PF is generally due to the large induction loads such as discharge lamps, induction motors, transformers etc. which take a lagging current and produce heat which returns to the generating station without doing any useful work as such it is essential to improve or correct the low PF so as to bring the current as closely in phase with the voltage as possible. That is the phase angle θ is made as small as possible. This is usually done by placing a capacitor load which produces a leading current.

The capacitor is to be connected in parallel with the inductive load.

R - C Series circuit

Objectives: At the end of this lesson you shall be able to
- **state the effect of frequency on capacitive reactance in R-C series circuit**
- **calculate power factor**
- **determine the power factor and phase angle**
- **state the R-C time constant while charging and discharging**

In a circuit with capacitance, the capacitive reactance (X_C) decreases when the supply frequency (f) increases as shown in Fig 1.



RELATION BETWEEN FREQUENCY AND CAPACITIVE REACTANCE

$$X_C \propto \frac{1}{f}$$

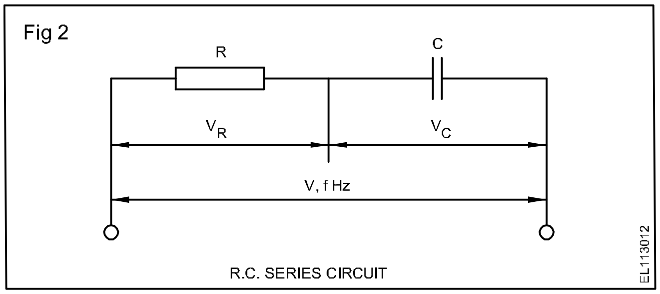
When the capacitive reactance X_C increases the circuit current decreases.

$$I \propto \frac{1}{X_C}$$

Therefore the increase in frequency (f) results in the increase of the circuit current in the capacitive circuit. When resistance (R), capacitance (C) and frequency f are known in a circuit, the power factor cos q can be determined as follows. (Fig 2)

$$X_C = \frac{1}{2\pi fC}$$

Fig 2



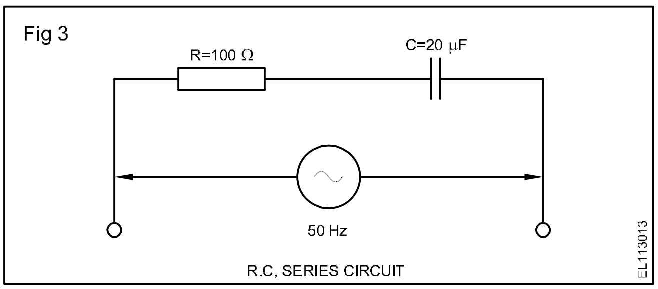
R.C. SERIES CIRCUIT

$$Z = \sqrt{R^2 + X_C^2}$$

Power factor, $\cos = \frac{R}{Z}$

Example 1: A capacitance of 20 µ f and a resistance of 100Ω are connected in series across a supply frequency of 50 Hz. Determine the power factor. (Fig 3)

Fig 3



R.C, SERIES CIRCUIT

Solution

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \frac{22}{7} \times 50 \times 20 \times 10^{-6}}$$

$$= \frac{7 \times 10^{-6}}{2 \times 22 \times 50 \times 20}$$

$$= \frac{7000000}{44000}$$

= 159.1 Ω, say 160 Ω.

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{10000 + 25600}$$

$$= \sqrt{36600} = 191.3$$

$$P.F. = \frac{R}{Z} = \frac{100}{191.3} = -.522$$

Capacitive reactance X_C in a capacitive circuit can be determined with the formula

$$X_C = \frac{1}{2\pi fC}$$

where X_C = capacitive reactance in ohm

f = frequency in Hz

C = Capacitance in farad

Power consumed in a R-C series circuit can be determined using the formula

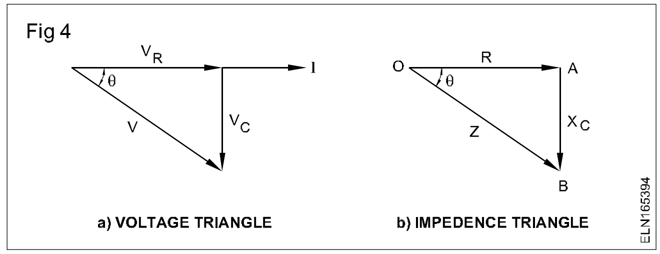
P = VI cos θ where P = power in watts

I = current in ampere

cos θ = power factor.

Vector diagram of voltages and their use to determine pf angle θ. (Fig 4)

Fig 4



$V_R = I_R$ drop across R (in phase with I)

$V_C = IX_C$ drop across capacitor (lagging I by 90°)

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$\therefore I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

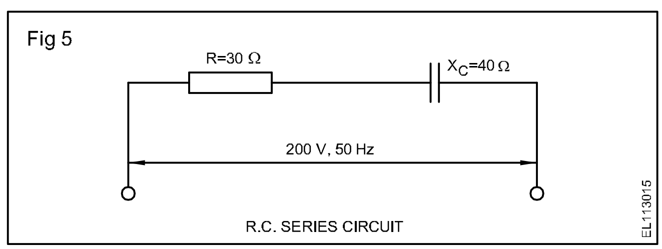
$\therefore Z = \sqrt{R^2 + X_C^2}$ where Z is the impedance of the circuit.

Power factor, cos θ = R/Z.

From pf cos θ the angle θ can be known referring to the Trignometric table.

Example 2: In RC series circuit shown in the diagram (Fig 5) obtain the following.

Fig 5



R.C. SERIES CIRCUIT

- Impedance in ohms
- Current in amps
- True power in watts
- Reactive power in var
- Apparent power in volt amp.
- Power factor

Solution

1 Impedence (Z)

$$= \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50$$

2 Current $I = \frac{V}{Z} = \frac{200}{50} = 4A$

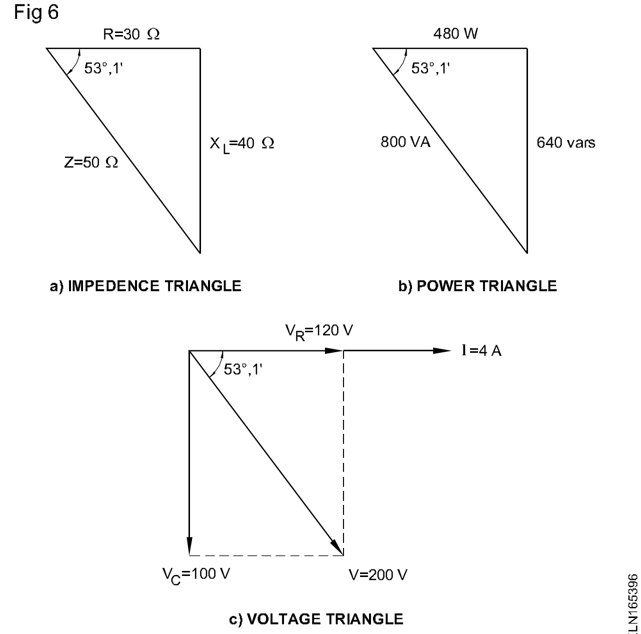
3 True power W = $I^2R = 4^2 \times 30 = 480W$
(Power consumed by capacitoir = zero)
$V_C = IX_C = 4 \times 40 = 160$ V

4 Reactive power VAR = $V_CI = 160 \times 4 = 640$ VAR
Apparent power VI = 200 x 4 = 800 VA

$$PF \quad \cos = \frac{R}{Z} = \frac{30}{50} = 0.6$$

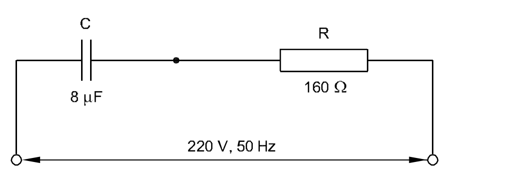
The impedance triangle, power triangle and voltage triangle for exercise 2 are showm in Fig 6

Fig 6



Example 3: An 8 µf capacitor is connected in series with an ohmic resistance of 160 Ω. A voltage of 220V AC, 50 Hz is applied to the circuit (Fig 7)

Fig 7



Calculate

a) the capacitive reactance

b) the impedance

c) the current

d) the active power

e) the reactive power.

Solution

a) $X_C = \frac{1}{2\pi fC} = \frac{10^6}{314 \times 8} = 400$

b) $Z = \sqrt{R^2 + X_C^2} = \sqrt{160^2 + 400^2}$

$= \sqrt{185600} = 430$

c) $I = \frac{V}{Z} = \frac{220}{430} = 0.51A$

d) W = $I^2R = 0.51^2 \times 160 = 41.62W$

e) VAR = V x I Sin θ = 220 x 0.51 x 0.9291

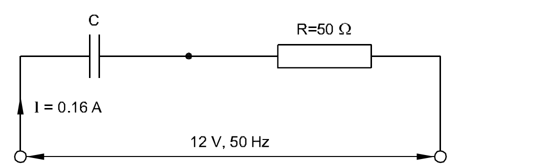
= 102.2 VAR

$$\text{Cos} = \frac{R}{Z} = \frac{160}{430} = 0.37$$

(θ = 18°18',Referring to the sine table,

sin θ = sin 18°18' = 0.9291.

Fig 8



Example 4: In the circuit shown in Fig 8, calculate a) the capacitive reactance and b) the capacitance of the capacitor.

Solution

$V_R = IR = 0.16 \times 50 = 8V$

$V_C = \sqrt{V^2 - V_R^2} = \sqrt{12^2 - 8^2} = \sqrt{80}$ = 9V (App)

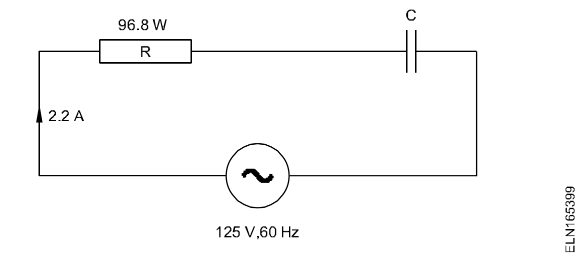
$$X_C = \frac{V}{I} = \frac{9}{0.16} = 56$$

$$X_C = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi fX_C} = \frac{1}{314 \times 56} = \frac{10^6}{314 \times 56} = 57\,\mu F$$

Example 5: A voltage of 125V at 60Hz is applied across a non-inductive resistance connected in series with a condenser. The current in the circuit is 2.2A. The power loss in the resistor is 96.8W and that in the condenser is negligible. Calculate the resistance and capacitance. (Fig 9)

Fig 9



Solution : Power loss I^2R = 96.8W

$$\therefore R = \frac{96.8}{I^2} = \frac{96.8}{2.2^2} = 20$$

Impedence $Z = \frac{V}{I} = \frac{125}{2.2} = 56.82$

Capacitance reactance $X_C = \sqrt{Z^2 - R^2}$

$$= \sqrt{56.82^2 - 20^2}$$

$= 53.2\Omega$

$X_C = 1 / (2\pi fC)$

$2\pi fC = 1 / X_C$

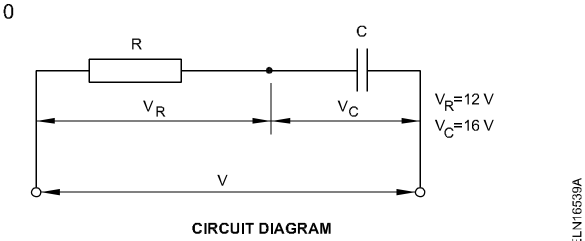
2 x 3.14 x 60 x C = 1/53.2

C = 1 / (53.2 x 2 x 3.14 x 60)

= 0.00005 F = 50 mF

Example 6: In the circuit shown (Fig 10)

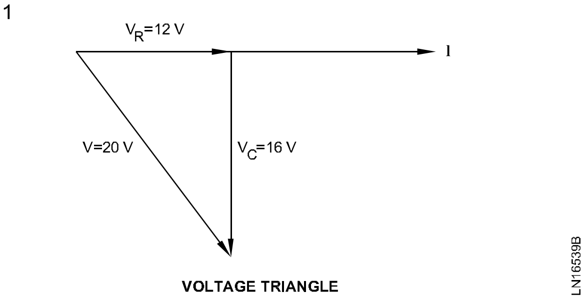
Fig 10



a) calculate the voltage V

b) draw the voltage triangle (Fig 11).

Fig 11



Solution

a) $V = \sqrt{V_R{}^2 + V_C{}^2} = \sqrt{12^2 + 16^2} = \sqrt{144 + 256}$

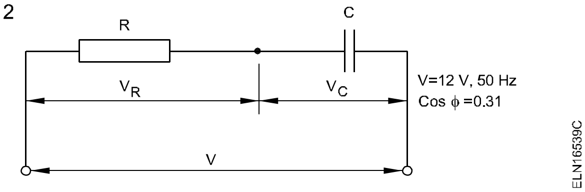
$= \sqrt{400}$ = 20V

Example 7: In the circuit shown, Fig 12 calculate

a) the resistor voltage

b) the capacitor reactance voltage.

Fig 12



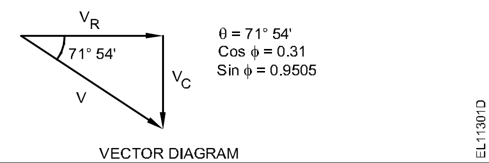
Solution

a) V_R = V cos θ = 12 x 0.31 = 3.72V

b) V_C = V sin θ = 12 x 0.9595 x 11.4V

Vector diagram Fig 13

Fig 13



'Time constant': One time constant is that time in seconds required for a completely discharged capacitor to charge 63% of the source voltage (charging voltage).

τ = R x C

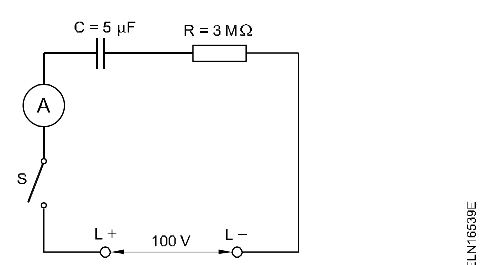
where τ = one time constant in seconds

R = resistance in ohms

C = capacitance in farad.

As shown in the (Fig 14), if

Fig 14



C= 5 μF, R = 3 M Ω, then

$\tau = 5 \times 10^{-6} \times 3 \times 10^{6}$ = 15 seconds

charging voltage = 100 V

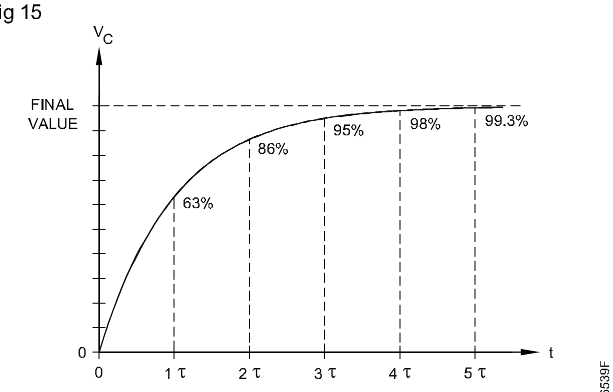
one time (τ) constant = 15 seconds. V_e= 63V

After 5 time constant a capacitor is 99.3% charged, for practical purposes it is taken that the capacitor is fully charged. A capacitor charges in 5 time constant.

A charging curve is shown in Fig 15.

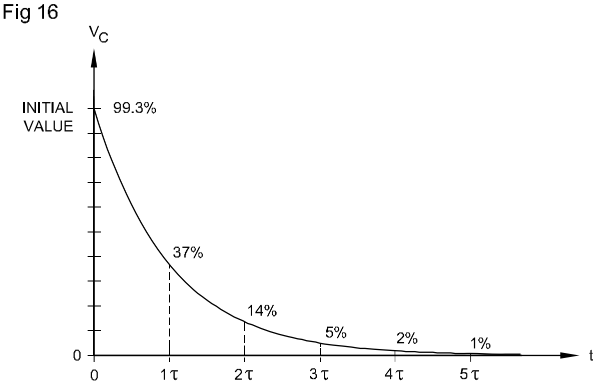
During discharging, a fully charged capacitor discharges in five time constant. A discharging curve is shown in Fig 16.

Fig 15



CHARGING CURVE

Fig 16



DISCHARGING CURVE

τ (sec)	Time taken (V)	**Charging voltage plates** (V)	**Voltage across the capacitor**
At the time of switching on		100	0
1τ	15	63% of 100 = 63	0 + 63 = 63
2τ	30	63% of (100-63) = 23.3 say 23	63 + 23 = 86
3τ	45	63% of (100-86) = 8.82 say 9	86 + 9 = 95
4τ	60	63% of (100-95) = 3.15 say 3	95 + 3 = 98
5τ	75	63% of (100-98) = 1.26 say 1.3	98 + 1.13 = 99.3

R L C series circuit

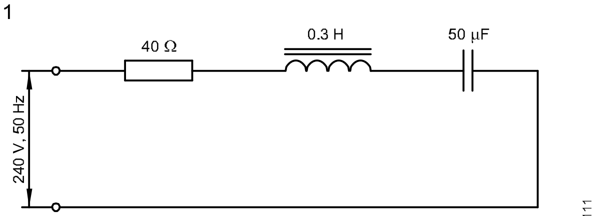
Objectives: At the end of this lesson you shall be able to

- **calculate the resultant reactance and impedance of the RLC series circuit**
- **state the impedance, voltage and power triangle.**
- **explain the necessary conditions for series resonance.**

Assume an AC single phase circuit consisting a resistance, inductor and capacitor in series. Various parameters could be calculated as shown in the example.

Example : The value of the components shown in Fig 1 is R = 40 ohms L = 0.3 H and C = 50µf. The supply voltage is 240 V 50 Hz. Calculate the inductive reactance, capacitance reactance, net reactance, impedance, current in the circuit, voltage drops across the R, L and C power factor, active power, reactive power and apparent power. Also draw the impedance triangle, voltage triangle and power triangle.

Fig 1



R L C SERIES CIRCUIT

Calculate the resultant reactance in RLC circuit : Inductance and capacitance have directly opposite effects in an AC circuit. The voltage drop caused by the inductive reactance of the coil leads the line current by 90°. The voltage drop across the inductor coil and the capacitor are 180 degrees apart and oppose each other. To calculate the net reactance in the above example:

Inductive reactance

$X = 2\pi fL = 314 \times 0.3 = 94.2\Omega$

Capacitive reactance

$$X = \frac{1}{2\pi fC} = \frac{1}{314 \times 0.00005} = \frac{1}{0.0157} = 63.69$$

Net reactance - X_L - X_C = 94.2 - 63.69 = 30.51Ω

Calculate the impedance: From the circuit given above the impedance can be found. The impedance is the resultant combination resistance and reactance. In this circuit, the impedance is the combination of the 40 ohms resistance and 30.51 Ω resultant reactance. The impedance for this circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + 30.51^2}$$

$$= \sqrt{1600 + 930.86} = \sqrt{2530.86} = 50.30$$

Draw the impedance triangle: Draw the horizontal line (X axis) indicating the circuit current.

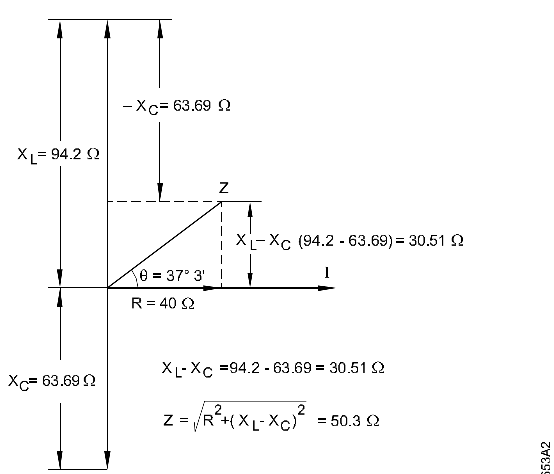
Draw along with current vector the value of R to a suitable scale i.e. 1cm = y ohm.

draw the vertical line perpendicular to the current vector in +y axis indicating the value of inductive reactance to the scale selected (1cm = y ohm)

draw a vertical the perpendicular to the current vector in _y axis indicating the value of capacitive reactance to the scale selected (1cm = y ohm).

Substract the value of X_C from, X_L as shown in Fig 2 the net reactance value is equal to 30.51 ohms. Complete the vectors by closing the parallelogram the reactance of the parallelogram is the impedance of the series RLC circuit.

Fig 2



IMPEDANCE TRIANGLE

Mathematically what we determined the values of net reactance and impedance could also be determined by the above vectorial method.

Measurement of current and voltage drop in RLC circuit. The voltage drop across R = E_R across L = E_L and drop across C = E_C and the formula for finding their values and given below.

$E_R = IR$

$E_L = IX_L$

$E_C = IX_C$

Current in given RLC series circuit: Current in this series circuit is I = E/Z = 240/50.3 = 4.77 amps.

Identifying whether the current flow is leading or lagging the voltage in a RLC series circuit: As this is a series circuit, the current is the same in all parts of the circuit, but the voltage drop across the resistor, the inductor coil and capacitor are

$E_R = IR$ = 4.77 x 40 = 190.8 volts

$E_L = IX_L$ = 4.77 x 94.2 Ω = 449.33 volts

$E_C = IX_C$ = 4.77 x 63.69 = 303.80 volts.

The vector sum of the voltage of 190.8 volts across the resistor and 145.53 volts across the net reactance of 30.51Ω is equal to the line voltage of 240 volts as shown below.

$$E = \sqrt{E^2R + (E_L - E_C)^2}$$

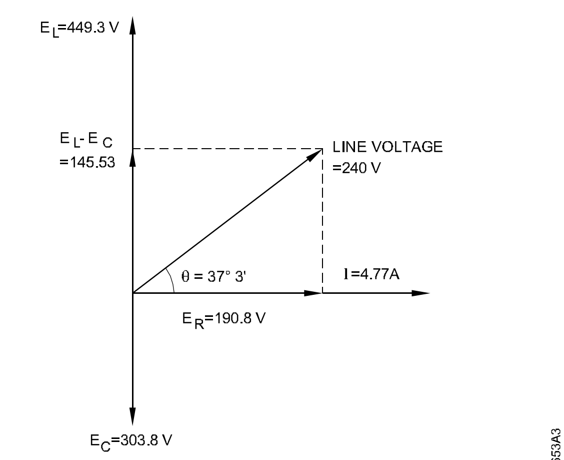
$$= \sqrt{190.8^2 + (449.33 - 303.80)^2}$$

$$= \sqrt{190.8^2 + 145.53^2}$$

E = 240 volts.

The voltage vector diagram could be drawn as shown in Fig 3.

Fig 3



VECTOR DIAGRAM

In this type of series circuit, the current is used as a horizontal reference line. The voltage value of 145.53 volts across the portion of the inductive reactance which is not cancelled out by the voltage across the capacitive reactance. The PF = E_R/E = 190.8/240 = 0.795 lag or PF = R/Z = 40/50.30 = 0.795 lag PF. In this circuit the phase angle is 37.3° lagging. This means that current lags the line voltage.

In an RLC series circuit, if X_L is greater then the voltage appearing across the inductor is high and that can be found out by IX_L. In the same way if the X_C value is greater in an RLC series circuit, the voltage appearing across the capacitor is more and can be found out by IX_C.

In the example given above, the voltage drop across resistance 40 Ω = 190.8 volts.

Voltage drop across inductance 0.3 H = 449.33 volts. Voltage drop across capacitor of 50 mf = 303.80 volts.

From these values it is clear that the voltage drop across the inductor and capacitor is higher than the supply voltage. Hence while connecting the meter to measure the voltage drop across inductor and capacitor it should be noted that the range of this should be high (in this case 0 - 500 volts)

Calculate the power factor: Power factor of the RLC series circuit can be found from the impedance triangle or voltage triangle as shown below

$$\text{Power factor} = \text{Cos} = \frac{R}{Z} \text{ or } \frac{E_R}{V}$$

$$\text{Power factor} = \frac{R}{Z} = \frac{40}{50.3} = 0.795$$

$$= \frac{E_R}{V} = \frac{190.8}{240} = 0.795$$

Calculate the active power (R_A): Active power can be calculated by using any one of the formulae given below

P = EI Cos θ = I^2R

= EI Cos θ = 240 x 4.77 x 0.795

= 910 walts

= $I^2R = 4.77^2 \times 40$

= 910 watts.

Calculate the reactive power P_q: Reactive power can be calculated using the formula

P_q = EI sin θ Vars

= 240 x 6.77 x 0.6074

= 695 Vars

Cos θ = 0.795

θ = 37°3'

Sin θ = Sin 37°3'

= 0.6074

Calculate the apparent power (P_{APP}): Apparent power can be calculated using the formula

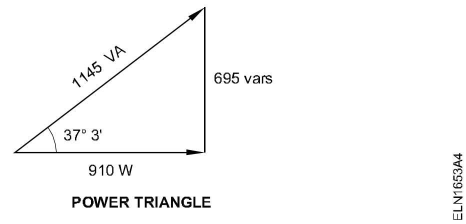
P_{APP} = EI volt-amperes

= 240 x 4.77

= 1145 Volt-amperes.

Draw the power triangle: Power triangle is shown in Fig 4.

Fig 4



POWER TRIANGLE

Resonance circuit: When the value of X_L and X_C are equal, the voltage drop across them will be equal and hence they cancel each other. The value of voltage drops V_L and V_C may be much higher than the applied voltage.

The impedance of the circuit will be equal to the resistance value. Full value of applied voltage appears across R and the current in the circuit is limited by the value of resistance only. Such circuits are used in electronic circuits like radio/TV turning circuits. When $X_L = X_C$ the circuit is said to be in resonance.

As current will be maximum in series resonant circuits it is also called acceptor circuits. For a known value of L and C the frequency at which this occurs is called as resonant frequency. This value can be calculated as follows when $X_C = X_L$

$$2\pi fL = \frac{1}{2\pi fC}$$

Hence resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

Power factor angle is commonly denoted by Theta θ. In some pages of this text it is denoted by Phi ϕ. As such these terms are used alternatively in this text.

Series resonance circuit

Objectives: At the end of this lesson you shall be able to
- **explain the impedance of series resonance circuit**
- **state the condition for series resonance and its expression**
- **state the resonance frequency and its formula**
- **state about the 'Q' factor (selectivity) of RLC circuit from graph.**

Series resonance circuit

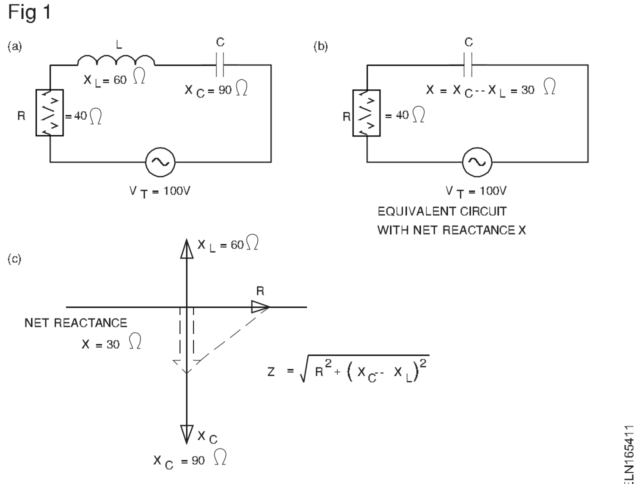
Impedance of series resonance circuit

A simple series LC circuit shown in Fig 1. In this series LC circuit,

- resistance R is the total resistance of the series circuit(internal resistance) in ohms,
- X_L is the inductive reactance in ohms, and
- X_C is the total capacitive reactance in ohms.

In the circuit at Fig 1a, since the capacitive reactance(90Ω) is larger than inductive reactance(60Ω), the net reactance of the circuit will be capacitive. This is shown in Fig 1b.

Fig 1



Note: If the capacitive reactance was smaller than inductive reactance the net reactance of the circuit would have been inductive.

All though the unit of measure of reactance and resistance is the same(ohms), the impedance, Z of the circuit is not given by the simple addition of R, X_L and X_C. This is because, X_L is +90° out of phase with R and X_C is -90° out of phase with R.

Hence the impedance Z of the circuit is the phasor addition of the resistive and reactive components as shown by dotted lines in Fig 1c. Therefore, Impedance Z of the circuit is given by,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

If X_L were greater than X_C, then the absolute value of impedance Z is will be,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For the circuit in Fig 2(a), total impedance Z is,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{40^2 + 30^2}$$

Z = 50Ω, Capacitive (because $X_C > X_L$)

Current I through the circuit is given by,

$$I = \frac{V}{Z} = \frac{100}{50} = 2 \text{ Amps}$$

Therefore, the voltage drop across the components will be,

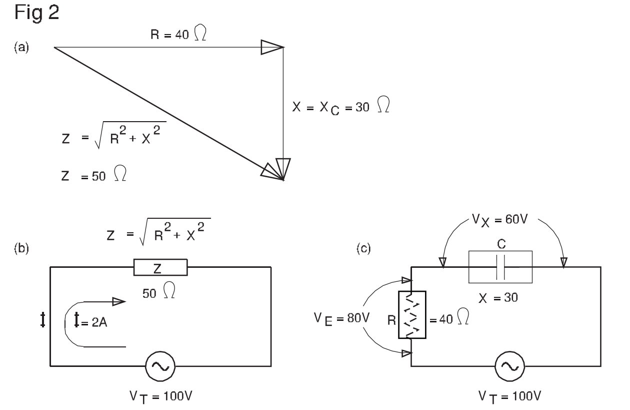
V_R = voltage drop across R = I.R = 2x40 = 80 volts

V_L = voltage drop across L = I.X_L = 2x60 = 120 volts

V_C = voltage drop across C = I.X_C = 2x90 = 180 volts.

Since V_L and V_C are of opposite polarity, the net reactive voltage V_X is = 180 - 120 = 60V as shown in Fig 2.

Fig 2



Note that the applied voltage is not equal to the sum of voltage drops across reactive component X and resistive component. This is again because the voltage drops are not in phase. But the phasor sum of V_R and V_X will be equal to the applied voltage as given below,

$$V_T = \sqrt{V_R^2 + V_X^2}$$

$$= \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{80^2 + 60^2} = 100 \text{ volts(applied voltage).}$$

Phase angle θ of the circuit is given by,

$$= \tan^{-1} \frac{X_C - X_L}{R}$$

Condition at which current through the RLC Series circuit is maximum

From the formula,

$Z = \sqrt{R^2 + (X_C - X_L)^2}$ it is clear that the total impedance Z of the circuit will become purely resistive when,

reactance $X_L = X_C$

In this condition, the impedance Z of the circuit will not only be purely resistive but also minimum.

Since the reactance of L and C are frequency dependent, at some particular frequency say f_r, the inductive reactance X_L becomes equal to the capacitive reactance X_C. In such a case, since the impedance of the circuit will be purely resistive and minimum, current through the circuit will be maximum and will be equal to the applied voltage divided by the resistance R.

Series resonance

From the above discussions it is found that in a series RLC circuit,

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Current $I = \frac{V}{Z}$

and,

Phase angle $= \tan^{-1} \frac{X_L - X_C}{R}$

If the frequency of the signal fed to such a series LC circuit (Fig 1a) is increased from 0 Hz, as the frequency is increased, the inductive reactance($X_L = 2\pi fL$) increases linearly and the capacitive reactance ($X_C = 1/2\pi fL$) decreases exponentially as shown in Fig 3.

As shown in Fig 3, at a particular frequency called the resonance frequency, f_r, the sum of X_L and X_C becomes zero($X_L - X_C = 0$).

From Fig 3 above, at resonant frequency ,

– Net reactance, X = 0 (i.e, $X_L = X_C$)

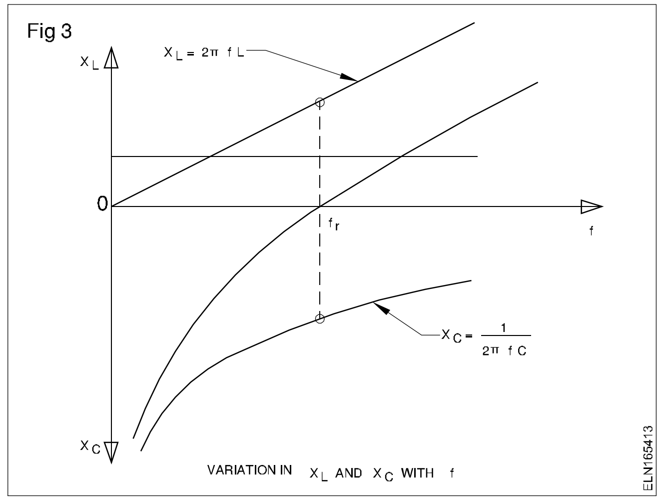
– Impedance of the circuit is minimum, purely resistive and is equal to R

– Current I through the circuit is maximum and equal to V/R

– Circuit current, I is in-phase with the applied voltage V (i.e. Phase angle = 0).

At this particular frequency f_r called resonance frequency, the series RLC is said to in a condition of series resonance.

Fig 3



VARIATION IN X_L AND X_C WITH f

Resonance occurs at that frequency when,

$X_L = X_C$ or $2\pi fL = 1/2\pi fC$

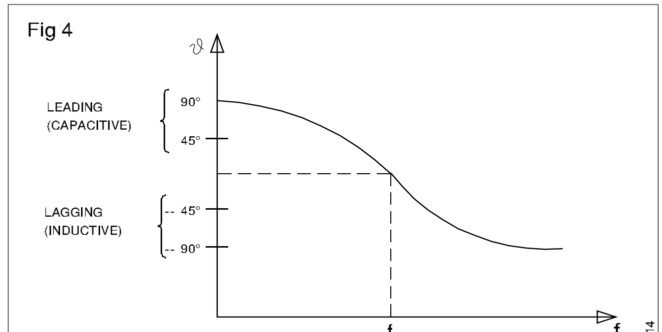
Therefore, **Resonance frequency, f_r** is given by,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \quad ....[1]$$

Reactance of series RLC above and below resonance frequency f_r

Fig 4 shows the variation of net reactance of a RLC circuit with the variation in frequency.

Fig 4



VARIATION OF NET REACTANCE

From Fig 4 above,it can be seen that the,

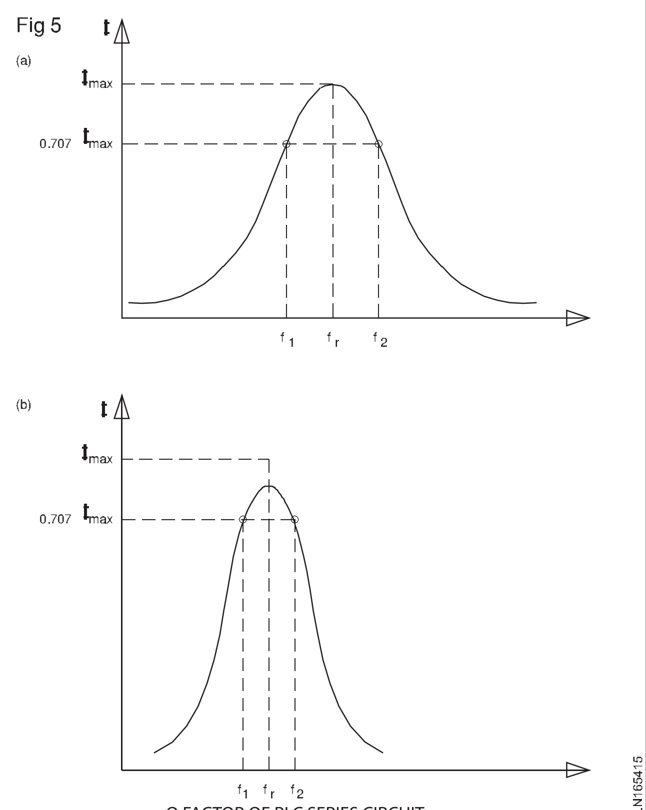
– net reactance is zero at resonant frequency f_r

– net reactance is capacitive below the resonant frequency f_r

– net reactance is inductive above the resonant frequency f_r.

Selectivity or Q factor of a series RLC circuit

Figs 5a and 5b two graphs showing the current through series two different RLC circuits for frequencies above and below f_r. f_1 and f_2 are frequencies at which the circuit current is 0.707 times the maximum current,I_{max} or the -3dB points.



Q FACTOR OF RLC SERIES CIRCUIT

Fig 5 indicates that series RLC circuits select a band of frequencies around the resonant frequency, f_r. This band(f_1 to f_2 is called the **band width** f of the series RLC circuit.

Bandwidth = $\Delta f = f_2 - f_1$ Hz.

where, f_2 is called the upper cut off frequency and f_1 is called the lower cut off frequency of the resonant circuit.

Comparing Figs 5a and 5b, it is seen that the bandwidth of 5b is smaller than that of 5a. This is referred to as the **selectivity** or **quality factor, Q** of the resonance circuit. The RLC circuit having the response shown in Fig 5b is more selective than that of Fig 5a. The quality factor, Q of a resonance circuit is given by,

$$\text{Quality factor} = Q = \frac{f_r}{f} = \frac{f_r}{f_2 - f_1} \qquad ...[2]$$

If Q is very large, the bandwidth f will be very narrow and vice-versa. The Q factor of the series resonance circuit depends largely upon the Q factor of the coil(inductance) used in the RLC circuit.

Therefore,

$$\text{Q of coil} = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

since,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Q of the series RLC circuit is given by,

$$Q = \frac{1}{R} \cdot \frac{\sqrt{L}}{\sqrt{C}} \qquad ...[3]$$

Application of series resonance circuits

A series resonance circuit can be used in any application where it is required to select a desired frequency. One such application is radio receiver.

R-L, R-C and R-L-C parallel circuits

Objectives: At the end of this lesson you shall be able to
- **explain the admittance triangle and the relationship between conductance, susceptance and admittance**
- **explain susceptance, conductance and admittance by symbols.**

R-L Parallel circuit

When a number of impedances are connected in parallel across an AC voltage, the total current taken by the circuit is the phasor sum of the branch currents (Fig 1).

There are two methods for finding the total current.

- Admittance method
- Phasor method

Admittance method

The current in any branch $I = \frac{E}{Z}$

$$= E \times \left|\frac{1}{Z}\right| \text{ where } \left|\frac{1}{Z}\right|$$

is called the **admittance** of the circuit i.e. admittance is the reciprocal of impedance. Admittance is denoted by 'Y' (Fig 2).

$$I = E \times \left|\frac{1}{Z}\right| = EY \quad \text{or} \quad Y = \frac{I}{E}$$

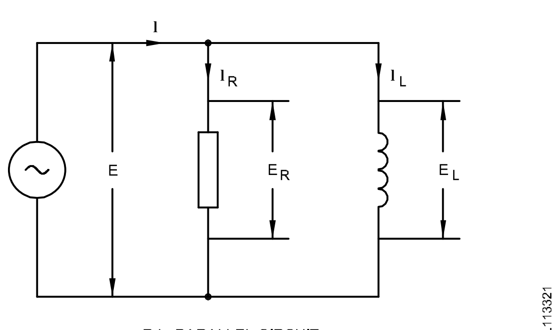
$$\therefore \text{Total admittance } (Y_T) = \frac{\text{total current}}{\text{common applied voltage}}$$

$$= \frac{\text{phasor sum of branch currents}}{\text{common applied voltage}}$$

= phase sum of separate admittance

Note: Supply voltage is referred as V or E interchangeably.

Fig 1

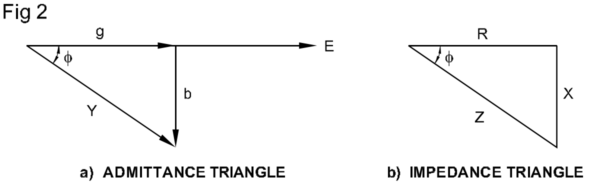


R.L. PARALLEL CIRCUIT

An admittance may be resolved into two components.

- A component in phase with the applied voltage called the conductance denoted by g.
- A component in quadrature (at right angle) with the applied voltage called **susceptance,** denoted by b.

Fig 2



a) ADMITTANCE TRIANGLE b) IMPEDANCE TRIANGLE

$$g = Y \cos\phi = \frac{1}{Z} \times \frac{R}{Z}$$

$$= \frac{R}{Z^2} = \frac{R}{R^2 + Z^2}$$

$$b = Y \sin\phi = \frac{1}{Z} \times \frac{X}{Z} = \frac{X}{Z^2}$$

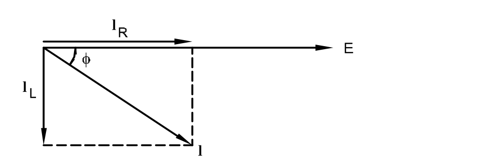
$$= \frac{X}{R^2 + X^2}$$

The unit of admittance, conductance and susceptance is called the mho symbol .

Relationship between branch current and supply voltage

In a R-L parallel circuit, the voltage across resistor (E_R) and inductor (E_L) are the same and equal to the supply voltage E. Hence E is the reference vector. The current through resistor (I_R) in phase with E_R is E. (Fig 3) The current through inductor (I_L) is lagging the E_L is E by 90^o. In short the current through resistor I_R is in phase and the current through inductor I_L, lags with applied voltage (E) by 90° . The power factor of R-L parallel circuit is cos ϕ where ϕ is the angle in between the total current and applied voltage.

Fig 3



R.L. PARALLEL CIRCUIT VECTOR

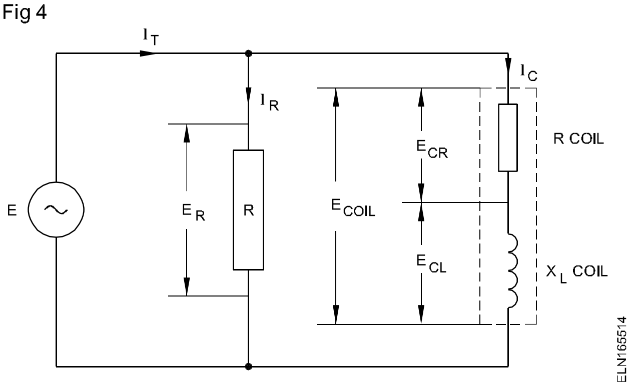
Representation of two branch currents and total current in the circuit containing R, R coil, X_L coil and supply voltage

I_R = branch current through resistor

I_C = branch effective current through coil

In a parallel circuit, the voltage across R (E_R) and across coil (E_C) is the same. Due to the applied voltage across coil (E_C) the coil current (I_C) flows through the coil. The current flowing through the coil is the effective current. The same current flows through the resistance and inductance of the coil (Fig 4).

Fig 4

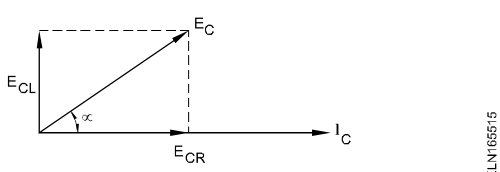


I_C = current flow through the resistance and inductance of the coil

E_{CR} = voltage drop in the coil due to resistance and in phase with I_C

E_{CL} = voltage drop in the coil due to inductance and leads the current by 90° (Fig 5).

Fig 5

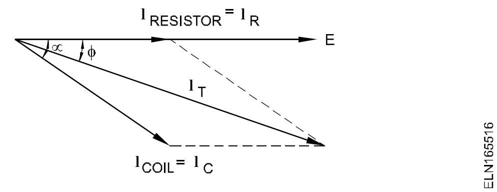


I_R = current through resistor in phase with E

I_C = current through coil lagging with E by α

I_T = total current lagging E by an angle ϕ.

Fig 6



The power factor of the above circuit is cos ϕ where ϕ is the phase angle between the applied voltage and total current (Fig 6).

Assignment

1 A coil of resistance 15 ohms and inductance 0.05H is connected in parallel with a non-inductive resistance of 20 ohms. Find (a) the current in each branch and (b) the phase angle between the total current of the whole arrangement and the applied voltage of 200 V at 50 Hz.

2 The load on a 250 V supply system is -

a) 12 A at 0.8 power factor lagging

b) 5 A at unity power factor.

Find the total load in kVA and its power factor.

3 The load on a 250 V supply system is -

a) 10 A at 0.5 power factor lagging

b) 5 A at unity power factor

c) 12 A at 0.866 power factor lagging.

Draw the vector diagram. Find the total load in kVA and its power factor.

4 A coil of resistance 15 ohms and inductance 0.05 H is connected in parallel with a non-inductive resistor of 40 ohms. Find the total current when a voltage of 200 V at 50 Hz. is applied. Give the phasor diagram.

AC Parallel circuit (R and C)

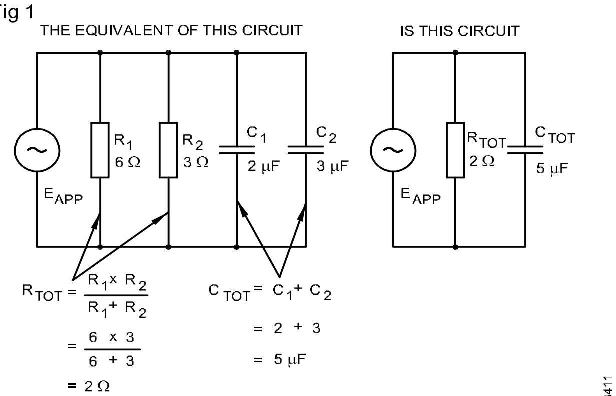
Objectives: At the end of this lesson you shall be able to

- **state the relationship between branch current, voltage in a parallel circuit**
- **solve problems in RC parallel circuit by admittance method**
- **compare the charecteristics of A.C series and parallel circuits**
- **state the R-L-C parallel circuit vector diagram**

Parallel RC circuits: In a parallel RC circuit, one or more resistive loads and one or more capacitive loads are connected in parallel across a voltage source. Therefore, resistive branches, containing only resistance and capacitive branches, containing only capacitance. (Fig 1) The current that leaves the voltage source divides among the branches; so there are different currents in different branches. The current is, therefore, not a common quantity, as it is in the series RC circuits.

Voltage: In a parallel RC circuit, as in any other parallel circuit, the applied voltage is directly across each branch. The branch voltages are, therefore, equal to each other, as well as to the applied voltage, and all three are in

Fig 1



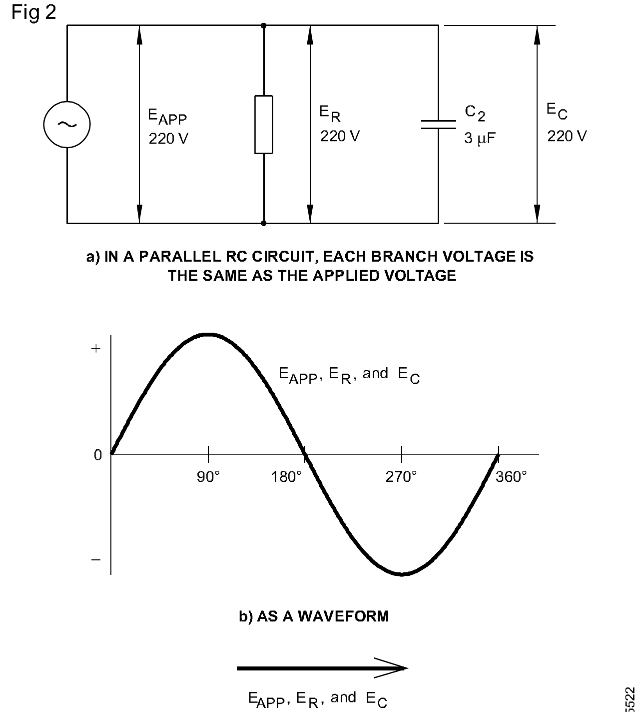
R.C. PARALLEL CIRCUITS

phase. (Fig 2) So if you know any one of the circuit voltages, you know all of them.

Since the voltage is common throughout the circuit, it serves as the common quantity in any vector representation of the parallel RC circuits. This means that on any vector diagram, the reference vector will have the same direction, or phase relationship, as the circuit voltage.

The two quantities that have this relationship with the circuit voltage, and whose vectors, therefore, have a direction of zero degrees, are the capacitor voltage and the current through the resistance.

Fig 2



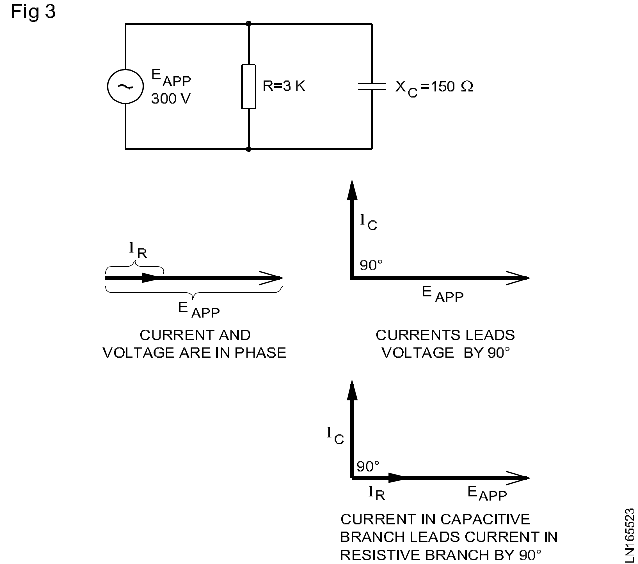
a) IN A PARALLEL RC CIRCUIT, EACH BRANCH VOLTAGE IS THE SAME AS THE APPLIED VOLTAGE

b) AS A WAVEFORM

c) AS A VECTOR

Branch current: The current in each branch of a parallel RC circuit is independent of the current in the other branches. The current within a branch depends only on the voltage across the branch, and the resistance or capacitive reactance contained in it. (Fig 3)

Fig 3

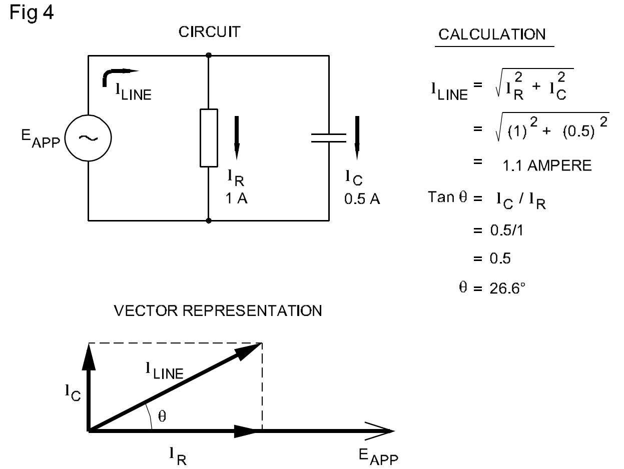


The current in the resistive branch is calculated from the equation: $I_R = E_{APP}/R$.

The current in the capacitive branch is found with the equation: $I_C = E_{APP}/X_C$.

The current in the resistive branch is in phase with the branch voltage, while the current in the capacitive branch leads the branch voltage by 90 degrees. Since the two branch voltages are the same, the current in the capacitive branch (I_C) must lead the current in the resistive branch (I_R) by 90 degrees. (Fig 4)

Fig 4



Line current: Since the branch currents in a parallel RC circuit are out of phase with each other, they have to be added vectorially to find the line current (Fig 5).

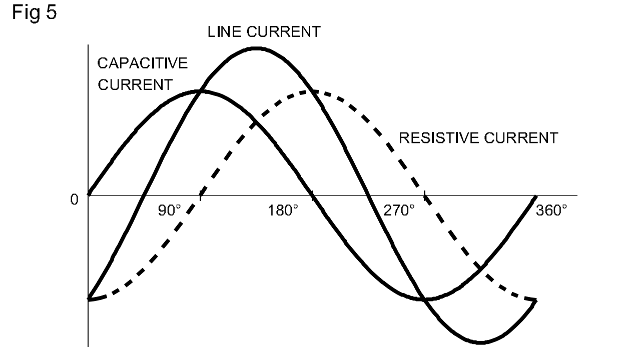
The two branch currents are 90 degrees out of phase, so their vectors form a right triangle, whose hypotenuse is the line current. The equation for calculating the line current, I_{LINE}, is

$$I_{LINE} = \sqrt{I_R^2 + I_C^2}$$

If the impedance of the circuit and the applied voltage are known, the line current can also be calculated from Ohm's Law.

$$I_{LINE} = \frac{E}{Z}$$

Fig 5



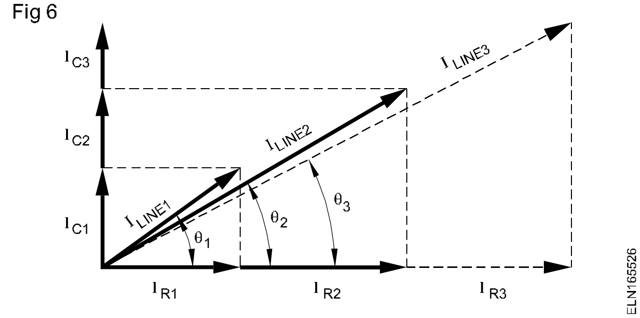
CAPACITIVE CURRENT LEADS THE RESISTIVE CURRENT BY 90°
THE LINE CURRENT IS THE VECTOR SUM OF THE RESISTIVE AND CAPACITIVE CURRENTS

In as much as the current in the resistive branch of a parallel RC circuit is in phase with the applied voltage, while the current in the capacitive branch leads the applied voltage by 90 degrees, the sum of the two branch currents, or line current, leads the applied voltage by some phase angle less than 90 degrees but greater than 0 degrees.

The exact angle depends on whether the capacitive current or resistive current is greater. If there is more capacitive current, the angle will be closer to 90 degrees; while if the resistive current is greater, the angle is closer to 0 degrees.

In cases where one of the currents is 10 or more times greater than the other, the line current can be considered to have a phase angle of 0 degrees if the resistive current is the larger (Fig 6). The value of the phase angle can be calculated from the values of the two branch currents with the equation:

Fig 6



$$\tan = \frac{I_C}{I_R}$$

By substituting the quantitites $I_C = E/X_C$ and $I_R = E/R$ in the above equation, two other useful equations for calcualting the phase angle, θ, can be derived, they are:

$$\tan = \frac{R}{E_C} \qquad \text{Cos} = \frac{R}{Z}$$

Once you know the line current and the applied voltage in a parallel RC circuit, you can find the circuit power using the same equations you learned for parallel RL circuits. These are:

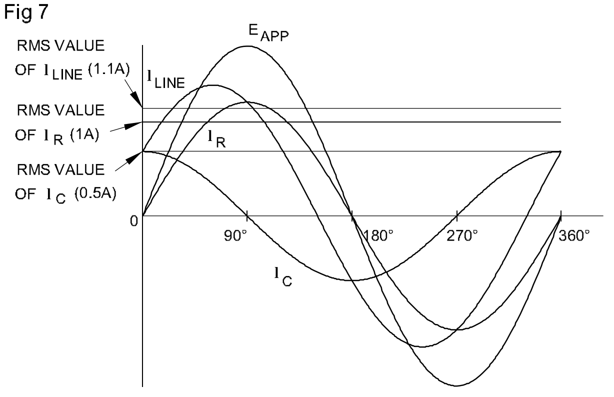
$$P_{APPARENT} = E_{APP} . I_{LINE}$$

$$P_{TRUE} = E_{APP} . I_{LINE} . \text{Cos } \theta$$

where cos θ is the power factor.

Current wave-forms: Since the branch currents in a parallel RC circuit are out of phase, their vector sum rather than their arithmetic sum equals the line current. This is the same condition that exists for the voltage drops in a series RC circuit. By adding the currents vectorially, you are adding their instantaneous values at every point, and then finding the average or effective value of the resulting current. This can be seen from the current wave-forms shown (Fig 7). They are the wave-forms for the circuit solved on the previous page.

Fig 7



EVERY POINT ON THE LINE CURRENT WAVEFORM (I_{LINE}) IS THE ALGEBRAIC SUM OF THE INSTANTANEOUS VALUES OF THE I_R AND I_C WAVEFORMS. THE RMS VALUE OF THE LINE CURRENT WAVEFORM IS THUS SHOWN TO BE EQUAL TO THE VECTOR SUM OF THE I_R AND I_C WAVEFORMS

Impedance: The impedance of a parallel RC circuit represents the total opposition to the current flow offered by the resistance of the resistive branch and the capacitive reactance of the capacitive branch. Like the impedance of a parallel RL circuit, it can be calculated with an equation that is similar to the one used for finding the total resistance of two parallel resistances.

However, just as you learned for parallel RL circuits, two vector quantities cannot be added directly, vector addition must be used. Therfore, the equation for calculating the impedance of a parallel RC circuit is

$$Z = \frac{RX_C}{\sqrt{R^2 + X_C^{\ 2}}}$$

where $\sqrt{R^2 + X_C^{\ 2}}$ is the vector addition of the resistance and capacitive reactance.

In cases where you know the applied voltage and the circuit line current, the impedance can be found simply by using Ohm's law in the form:

$$Z = \frac{E_{APP}}{I_{LINE}}$$

The impedance of a parallel RC circuit is always less than the resistance or capacitive reactance of the individual branches.

The relative values of X_C and R determine how capacitive or resistive the circuit line current is. The one that is the smallest, and therefore, allows more branch current to flow, is the determining factor.

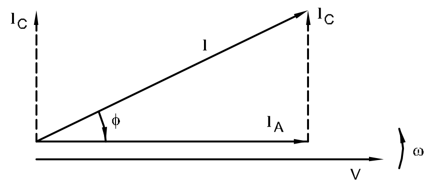
Thus, if X_C is smaller than R, the current in the capacitive branch is larger than the current in the resistive branch, and the line current tends to be more capacitive.

The opposite is true if R is smaller than X_C. When X_C or R is 10 or more times greater than the other, the circuit will operate for all practical purposes as if the branch with the larger of the two did not exist.

RC Parallel circuit - Admittance method

Admittance: In order to derive the admittance of a parallel circuit consisting of a resistance R and a capacitive reactance X_C we use the phasor diagram representation of the currents, I_A, I_C and I (Fig 1), and the corresponding admittance triangle (Fig 2).

Fig 1



PHASOR DIAGRAM

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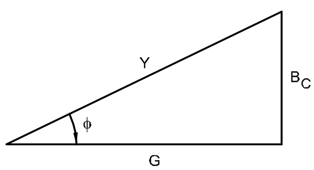
Fig 2



ADMITTANCE TRIANGLE

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Admittance $Y = \sqrt{G^2 + B_C{}^2}$

Total current I = VY

The current triangle gives

Active current $I_A = I \cos \theta$

Reactive current $I_C = I \sin \theta$

Total current $I = \sqrt{I_A^2 + I_C^2}$

From both triangles, the phase relationship is given by

$$\tan \phi = \frac{I_C}{I_A} = \frac{B_C}{G} = \frac{R}{X_C}$$

We can derive the values of R and X_C if we know the voltage V, the current I and the phase angle ϕ.

$Y = \frac{V}{I}$ conductance $G = Y \cos \phi$

susceptance $B_C = Y \sin \phi$

$R = \frac{1}{G}$ and $X_C = \frac{1}{B_C}$.

Parallel connection of R and X_C (Fig 3)

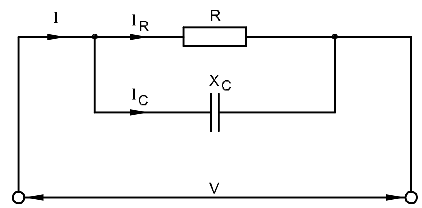
Graphic solution:

1 V as common vector

2 I_R in phase with V

3 I_C leads by 90°

Fig 3



R.C. PARALLEL CIRCUIT

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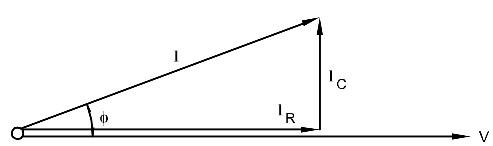
4 I as resultant (Fig 4)

5 f between V and I.

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C}\right)^2}$$

$$Y = \sqrt{G^2 + B_C^2} \text{ (Refer Fig 4)}$$

Fig 4



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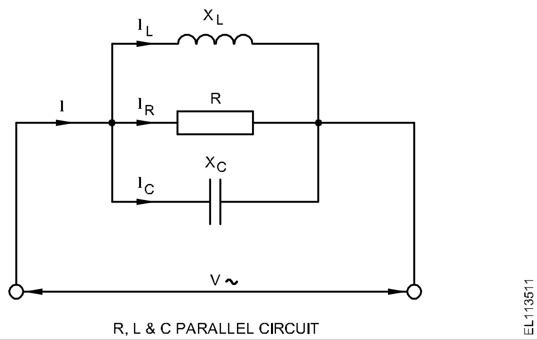
Comparison of series and parallel RC circuits

Quantity	Series RC circuit	Parallel RC circuit
Current	It is the same everywhere in circuit. Currents through R and C are, therefore, in phase.	It divides between resistive and capacitive branches. $I_{TOT} = \sqrt{I_R^{\,2} + I_C^{\,2}}$ $I_R = \frac{E_{APP}}{R}$ $I_C = \frac{E_{APP}}{X_C}$ Current through C leads current through R by 0°
Voltage	Vector sum of voltage drops across R and C equals applied voltage (Fig 5). $E_{APP} = \sqrt{E_R^{\,2} + E_C^{\,2}}$ Voltage across C lags voltage across R by 90°.	Voltage across each branch is the same as applied voltage. Voltages across R and C are, therefore, in phase. $E_R = E_C = E_{APP}$
Impedance	It is the vector sum of resistance and capacitive reactance. used. $Z = \sqrt{R^2 + X_C^{\,2}}$	It is calculated the same way as parallel resistances are, except that vector addition is $Z = \frac{RX_C}{\sqrt{R^2 + X_C^{\,2}}}$
Phase angle θ	It is the angle between the circuit current and the applied voltage. $\tan = \frac{E_C}{E_R} = \frac{X_C}{R}$ $\cos = \frac{R}{Z}$	It is the angle between line current and applied voltage. $\tan = \frac{I_C}{I_R} = \frac{R}{X_C}$ $\cos = \frac{R}{Z}$
Power	Power delivered by source is apparent power. Power actually consumed in the circuit is true power. Power factor determines what portion of apparent power is true power. $P_{APP} = E_{APP}I$	$P_{TRUE} = E_{APP}I \cos\theta$ P.F. = $\cos\theta$
Effect of increasing frequency	X_C decreases, which in turn causes the circuit current to increase. Phase angle decreases,which means that the circuit is more resistive.	X_C drecreases, the capacitive branch current increases, and so line current also increases. phase angle increases, which means that the circuit is more capacitive.
Effect of increasing resistance	Phase angle decreases, which means that the circuit is more resistive	Phase angle increases, which means that the circuit is more capactive
Effect of increasing capacitance	Phase angle decreases, which means that the circuit is more resistive	Phase angle increases, which means that the circuit is more capactive

R, L and C Parallel circuit - Vector diagram

Parallel connection of R, X_L and X_C: X_L and X_C oppose each other, that is to say, I_L and I_C are in opposition, and partly oppose one another (Fig 1).

Fig 1



R, L & C PARALLEL CIRCUIT

$I_X = I_C - I_L$ or $I_L - I_C$, depending on whether the capacitive or inductive current dominates.

Graphic solution: when $I_L > I_C$

1 V as common value

2 I_R in phase with V

3 I_C leads by 90°

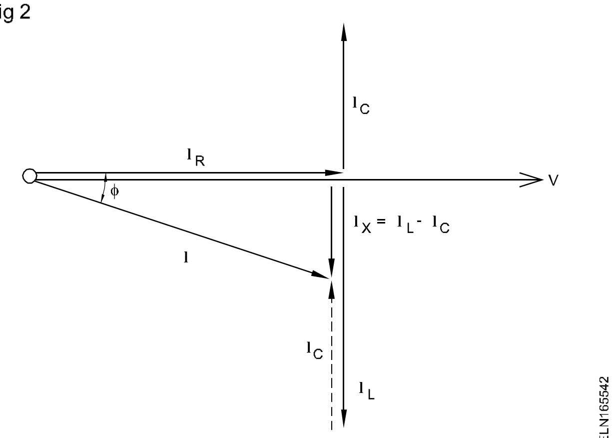
4 I_L lags by 90°

5 $I_X = I_L - I_C$

6 I as resultant

ϕ in this case inductive, I lags (Fig 2)

Fig 2



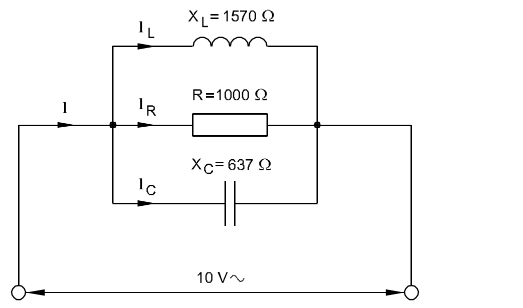
Particular case: X_L and X_C are equally large - I_L and I_C cancel each other. Z = R; parallel resonance occurs.

Currents in the reactances may be greater than the total current.

The calculation of the resonant frequency is the same as for the series connection.

Example: Calculate the value of I_T Z power factor and power for the circuit in Fig 3.

Fig 3



Given

$V_T = 10V$

$R = 1000\ \Omega$

$X_L = 1570\ \Omega$

$X_C = 637\ \Omega$

Known: Ohm's Law

$$I_T = \sqrt{(I_C - I_L)^2 + I_R^{\ 2}}$$

Solution

$$I_C = \frac{10\ V}{637} = 0.0157\ A = 15.7\ mA$$

$$I_L = \frac{10\ V}{1570} = 0.0064\ A = 6.4\ mA$$

$$I_R = \frac{10\ V}{1000} = 0.01 = 10\ mA$$

$$I_T = \sqrt{(0.0157 - 0.0064)^2 + (0.01)^2}$$

$= 0.0137A = 13.7\ mA$

$$Z = \frac{10V}{0.0137\ A} = 730$$

$$P.F = \frac{Z}{R} \quad Y = \frac{1}{Z} \quad \text{and} \quad g = \frac{1}{R}$$

$$= \frac{730}{1000} = 0.73$$

$$= \frac{g}{Y} = \frac{1}{R} \times \frac{1}{1/Z} = \frac{Z}{R}$$

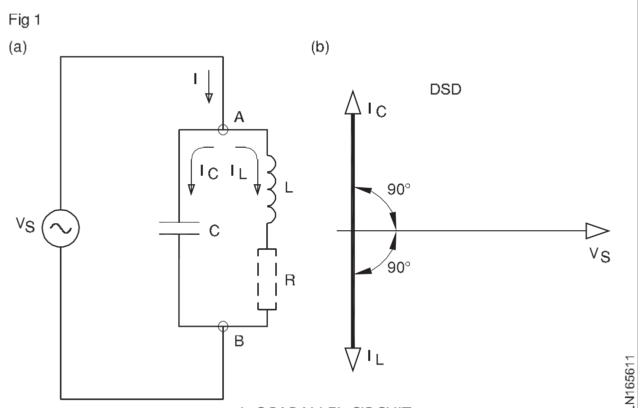
Parallel resonance circuits

Objectives: At the end of this lesson you shall be able to
- **state the characteristics of R-L-C parallel circuits at resonance**
- **explain the term band-width in parallel LC circuits**
- **explain the storage action in parallel LC circuits**
- **list a few applications of parallel LC circuits**
- **compare the properties of series and parallel resonance circuits**

Parallel resonance

The circuit at Fig 1, having an inductor and a capacitor connected in parallel is called parallel LC circuit or parallel resonance circuit. The resistor R, shown in dotted lines indicate the internal DC resistance of the coil L. The value of R will be so small compared to the inductive reactance, that it can be neglected.

From Fig 1a, it can be seen that the voltage across L and C is same and is equal to the input voltage V_S.



L-C PARALLEL CIRCUIT

By Kirchhoff's law, at junction A,

$I = I_L + I_C$.

The current through the inductance I_L (neglecting resistance R), lags V_S by 90°. The current through the capacitor I_C, leads the voltage V_S by 90°. Thus, as can be seen from the phasor diagram at Fig 1b, the two currents are out of phase with each other. Depending on their magnitudes, they cancel each other either completely or partially.

If $X_C < X_L$, then $I_C > I_L$, and the circuit acts capacitively.

If $X_L < X_C$, then $I_L > I_C$, and the circuit acts inductively.

If $X_L = X_C$, then $I_L = I_C$, and hence, the circuit acts as a purely resistive.

Zero current in the circuit means that the impedance of the parallel LC is infinite. This condition at which, for a particular frequency, f_r, the value of $X_C = X_L$, the parallel LC circuit is said to be in parallel resonance.

Summarizing, for a parallel resonant circuit, at resonance,

$X_L = X_C$,

$Z_p = \infty$

$I_L = I_C$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$I = \frac{V}{Z_P} \approx 0$$

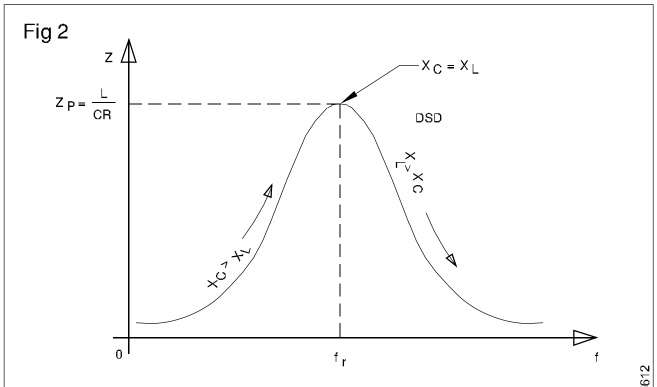
In a parallel resonance circuit, with a pure L(no resistance) and a pure C(loss-less), at resonance the impedance will be infinite. In practical circuits, however small, the inductor will have some resistance. Because of this, at resonance, the phasor sum of the branch currents will not be zero but will have a small value I.

This small current I will be in phase with the applied voltage and the impedance of the circuit will be very high although not infinite.

Summarizing, the three main characteristics of parallel resonance circuit at resonance are,

- phase difference between the circuit current and the applied voltage is zero
- maximum impedance
- minimum line current.

The variation of impedance of a parallel resonance circuit with frequency is shown in Fig 2.



FREQUENCY VERSES IMPEDENCE OF A PARALLEL RESONANT CIRCUIT

In Fig 2, when the input signal frequency to the parallel resonance circuit is moved away from resonant frequency f_r, the impedance of the circuit decreases. At resonance the impedance Z_p is given by,

$$Z_P = \frac{L}{CR}$$

At resonance, although the circuit current is minimum, the magnitudes of I_L & I_C will be much greater than the line current. Hence, a parallel resonance circuit is also called current magnification circuit.

For further details on current magnification in parallel resonance refer reference books at the end of this book.

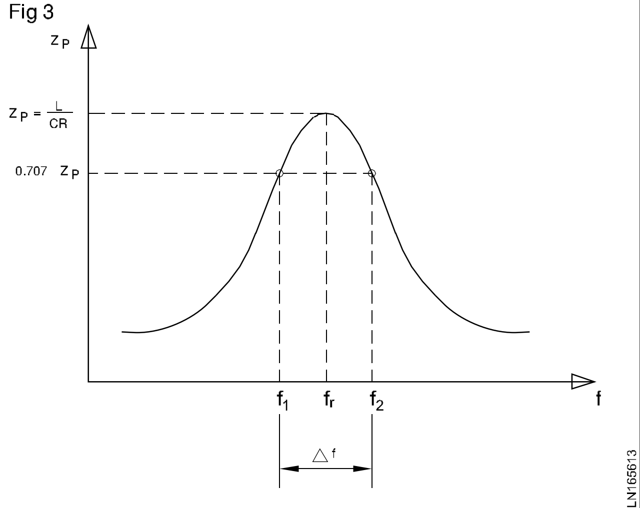
Bandwidth of parallel resonant circuits

As discussed in series resonance, all resonant circuits have the property of discriminating between the frequency at resonance(f_r), and those not at resonance. This discriminating property of the resonant circuit is expressed in terms of its **bandwidth(BW).**

In the case of series resonant circuits the response of the circuit at resonance frequency(f_r) is in terms of the line current(which is maximum), and in a parallel resonant circuit, it is in terms of the impedance(which is maximum).

The bandwidth of a parallel resonant circuit is also defined by the two points on either side of the resonant frequency at which the value of impedance Z_p drops to 0.707 or $1/\sqrt{2}$ of its maximum value at resonance, as shown Fig 3.

Fig 3



From Fig 3, the bandwidth of the parallel resonance circuit is,

Bandwidth, BW = $\Delta f = f_2 - f_1$

As can be seen in Fig 3, the value of Z_p is dependent on the resistance R of the coil (Z_p = L/CR). If R is less Z_p will be larger and vice versa.

Since the bandwidth depends on Z_p and Z_p depends on R, we can say that the bandwidth of a resonant circuit depends upon the resistance associated with the coil. The resistance of the coil in turn decides the Q of the circuit. Thus, the Q of the coil decides the band width of the resonant circuit and is expressed as,

$$\text{Bandwidth(BW)} = (f_2 - f_1) = \frac{f_r}{Q}$$

Storage action of parallel resonance circuit

At parallel resonance, though the circuit current is minimum(ideally zero), I_L and I_C will still be there. This I_L and I_C will be a circulating current in the closed loop formed by L and C.

This circulating current will be very high at resonance. This circulating current flip-flops between the capacitor and inductor, alternately charging and discharging each. When a capacitor or an inductor is charged, it stores energy. When it is discharged it gives up the energy stored in it. The current inside the LC circuit switches the stored energy back and forth between L and C. If the inductor had no resistance and if the capacitor was loss-free, then, no more external energy would be required to retain this flip-flop or oscillation of charging and discharging.

But, in a practical circuit, since ideal L and C cannot be obtained, some amount of the circulating energy is lost due to the resistance of the coil and the loss due to capacitor. This lost energy is the only energy the power supply source(V_S) must supply in the form of circuit current,I.

This current, therefore, is called as **make-up current.** It is this storage action of the parallel-resonant circuit which gives rise to the term **tank circuit**, often used with parallel resonant circuits. Hence, parallel resonant circuits are also called tank circuits.

Application of parallel resonant circuits

Parallel resonance circuits or tank circuits are commonly used in almost all high frequency circuits. Tank circuits are used as collector load in class-C amplifiers instead of a resistor load as shown in Fig 4.

Fig 4

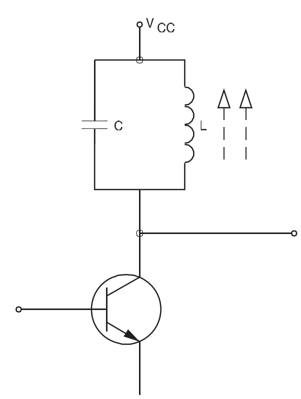


Table below gives a comparison between series resonant and parallel resonant circuit at frequencies above and below their resonant frequency f_r.

Property	Series circuit	Parallel circuit
	At resonant frequency	
Resonant frequency, f_r	$= \frac{1}{2\pi\sqrt{LC}}$	$= \frac{1}{2\pi\sqrt{LC}}$
Reactance	$X_L = X_C$	$X_L = X_C$
Impedance	Minimum (Z_r = R)	Maximum (Z_r = L/CR)
Current	Maximum	Minimum
Quality factor	$\frac{X_L}{R}$	$\frac{X_L}{R}$
Bandwidth	$\frac{X_L}{R}$	$\frac{X_L}{R}$
	Above resonant frequency	
Reactance	$X_L > X_C$	$X_C > X_L$
Impedance	Increases	Decreases
Phase difference	The current lags behind the applied voltage.	The current leads the applied voltage.
Type of reactance	Inductive	Capacitive
	Below resonant frequency	
Reactance	$X_C > X_L$	$X_L > X_C$
Impedance	Increases	Decreases
Phase difference	The current leads the applied voltage.	The current lags behind the applied voltage.
Type of reactance	Capacitive	Inductive

Power, energy and power factor in AC single phase system - Problems

Objectives: At the end of this lesson you shall be able to
- **state the relationship between power and power factor in single phase circuits**
- **state the connection diagram for measuring power factor using a direct reading meter.**
- **calculate the problem related to P.F and power in A.C circuits**

The power in a DC circuit can be calculated by using the formulae.

- P = E x I watts
- P = E^2/R watts.

The use of the above formulae in AC circuits will give true power only if the circuit contains pure resistance. Note that the effect of reactance is present in AC circuits.

Power in AC circuit: There are three types of power in AC circuits.

- Active power (True power)
- Reactive power
- Apparent power

Active power (True power)**:** The calculation of active power in an AC circuit differs from that in a direct current circuit. The active power to be measured is the product of V x I x Cos θ where Cos θ is the power factor (cosine of the phase angle between current and voltage). This indicates that with a load which is not purely resistive and where the current and voltage are not in phase, only that part of the current which is in phase with the voltage will produce power. This can be measured with a wattmeter.

Reactive power (P_r): With the reactive power (wattless power)

$$P_r = V \times I \times \sin\theta$$

only that part of the current which is 90° out of phase (90° phase shift) with the voltage is used in this case. Capacitors and inductors, on the other hand, alternatively store energy and return it to the source. Such transferred power is called reactive power measured in volt/ampere reactive or vars. Unlike true power, reacitve power can do no useful work.

Apparent power: The apparent power, P_a = V x I.

The measurement can be made in the same way as for direct current with a voltmeter and ammeter.

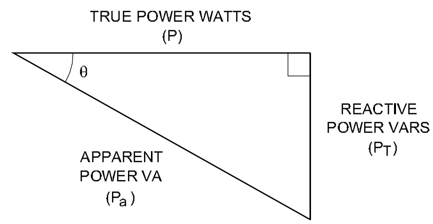
It is simply the product of the total applied voltage and the total circuit current and its unit is volt-ampere (VA).

The power triangle: A power triangle identifies three different types of power in AC circuits.

- True power in watts (P)
- Reactive power in vars (P_r)
- Apparent power VA (P_a)

The relationship among the three types of power can be obtained by referring to the power triangle. (Fig 1)

Fig 1



POWER TRIANGLE

Therefore

$P_a^2 = P^2 + P_r^2$ volt-amperes (VA)

where `P_a' is the apparent power in volt-ampere (VA)

`P' is the true power in watts (W)

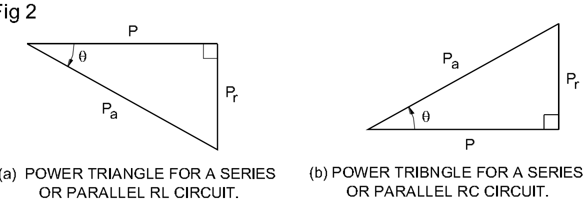
P_q is the reactive power in volt-amperes reactive. (VAR)

Power factor: The ratio of the true power delivered to an AC circuit compared to the apparent power that the source must supply is called the power factor of the load. If we examine any power triangle (Fig 2), you may see the ratio of the true power to the apparent power is the cosine of the angle θ.

$$\text{Power factor} = \frac{P}{P_a} = \text{Cos}$$

From the equation, you can observe that the three powers are related and can be represented in a right-angled power triangle, from which the power factor can be obtained as the ratio of true power to apparent power. For inductive loads, the power factor is called lagging to distinguish it from the leading power factor in a capacitive load. (Fig 2)

Fig 2



(a) POWER TRIANGLE FOR A SERIES OR PARALLEL RL CIRCUIT.

(b) POWER TRIBNGLE FOR A SERIES OR PARALLEL RC CIRCUIT.

POWER TRIANGLES SHOWING THAT $\frac{P}{P_a}$=COS θ

A circuit's power factor determines how much current is necessary from the source to deliver a given true power. A circuit with a low power factor requires a higher current than a unity power factor circuit.

Single phase energy

The product of true power and time is known as energy.

(ie) Energy = T.Power x time

= Voltage x current x power factor x time

= VI Cos θ x t (time is in hour)

The unit of energy is watt hour and commercial unit is represented in 'KWH' (or) unit. (Board of trade unit. B.O.T)

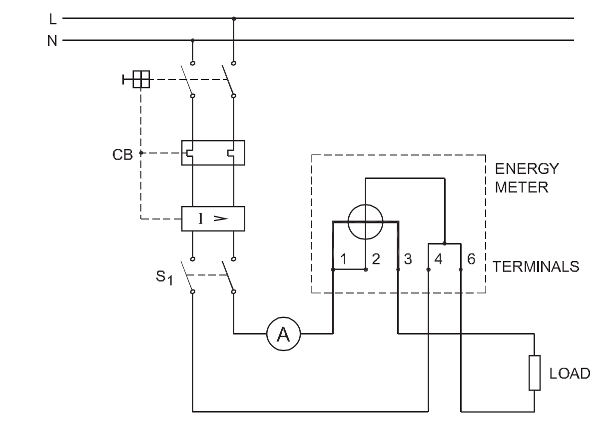
The energy depends upon the following factors:

- Voltage
- Current
- Power factor (load)
- Time

Single phase energy can be measured by energy meter. It contains 4 terminals (Incoming 2 and outgoing 2 common neutral)

The connection is shown in Fig 3.

Fig 3

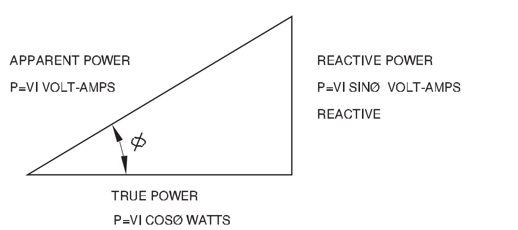


SINGLE PHASE ENERGY METER CONNECTION

Power in AC circuit having R L and C in series

As we have already studied, the power triangle has three components as shown in Fig 1.

Fig 1



POWER TRIANGLE

The above formula could be used in any AC single phase circuit. But the value of capacitive reactance and the inductive reactance decides whether the circuit is capacitive or inductive. When the value of the capacitive reactance is more than the value of inductive reactance, the PF will be leading or vice versa.

A series AC circuit consisting of 100 ohms, an inductance of 0.2 H and a capacitance of 120 μF are connected across 200 V 50c/s. Calculate the impedance, current, power factor and power absorbed.

The capacitive reactance = 1/ 2πfc ohms.

$$X_C = \frac{1\times10^6}{2\times\pi\times50\times120} = 26.53 \text{ ohms}$$

The inductive reactance

X_L = 2πfL.

X_L = 2 x π x 50 x 0.2 = 62.83 ohms.

Therefore, $X_L - X_C$ = 62.83 – 26.53 = 36.30 ohms.

The impedance = $\sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{100^2 + (62.83 - 26.53)^2}$$

$$= \sqrt{100^2 + (36.3)^2} = 106.4 \text{ohms}$$

$$\text{The current} = \frac{\text{Voltage}}{\text{Impedence}} = \frac{200}{106.4} = 1.88\text{A}$$

$$\text{The power factor} = \frac{R}{Z} = \frac{100}{106.4} = 0.94 \text{ (lagging)}$$

As the value of X_L is greater than that of X_c the circuit is having a lagging PF.

The power absorbed = V I cos θ

= 200 x 1.88 x 0.94 = 353.4 W.

Example 1

Calculate the current and its power factor when a resistance of 10 ohms, an inductance of 0.1 H and a condenser of 100μF capacitance are connected in series across 220 V 50 c/s supply mains.

Solution

R = 10 ohms

L = 0.1 H

C = 100 μF

X_C = 1/ 2πfC

$$X_C = \frac{10^6}{2 \times 3.14 \times 50 \times 100}$$

= 31.85 ohms.

$X_L = 2\pi FL$

= 2 x 3.14 x 50 x 0.1

= 31.4 ohms.

$X = X_C - X_L = 31.85 - 31.4$

= 0.45 ohms.

$Z = \sqrt{10^2 + (0.45)^2}$ = 10 ohms (approx.)

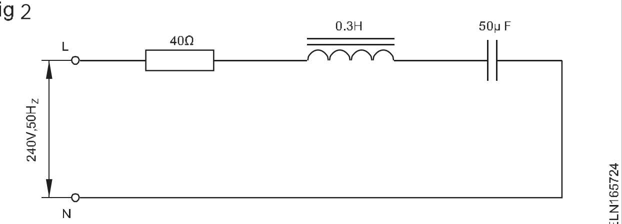
I = 220/ 10 = 22 A

PF = cos θ = R/Z = 10/10 = 1.Unity PF approx.

Example 2

In the circuit given in Fig 2.

Fig 2



Calculate:-

a the resulting reactance

b the impedance

c the current

d voltage drop across R,L&C

e draw the vector diagram

f Compare the calculated supply voltage with the applied supply voltage

g power factor

h power factor angle.

Solution

a Inductive reactance

$X_L = 2\pi fL$ = 2 x 3.14 x 50 x 0.3 = 94.2 ohms

$X_C = 1/2\pi fC$

$$X_C = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = \frac{10^6}{15714} = 63.69 \text{ ohms}$$

Net reactance = $X_L - X_C$ = 94.2 – 63.69 = 30.51 ohms.

The impedance for this circuit is Z

b $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (30.51)^2}$

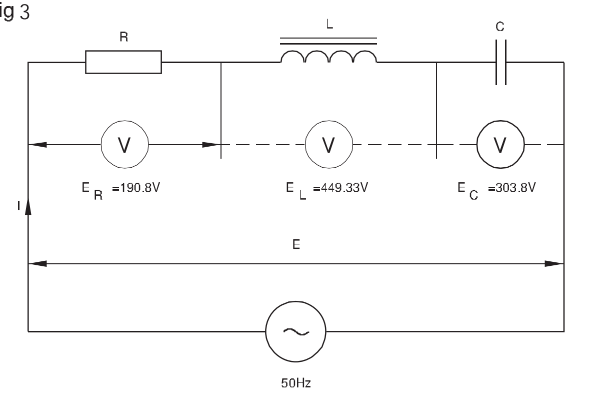
$= \sqrt{1600 + 930.86} = \sqrt{2530.86} = 50.30$ ohms

c Current in given RLC series circuit

Current in this series circuit is I = E/ Z = 240/50.3= 4.77 amps.

d Voltage drop across R, L and C. (Fig 3)

Fig 3



E_R = IR = 4.77 X 40 = 190.8 volts

$E_L = IX_L$ = 4.77 X 94.2 ohms = 449.33 volts

$E_C = IX_C$ = 4.77 X 63.69 = 303.80 volts.

The vector sum of the voltage of 190.8 volts across the resistor and the difference between the drops across inductor and the capacitor ($E_L - E_C$) 145.53 volts is equal to the line voltage of 240 volts where

$$E = \sqrt{E_R^{\;2} + (E_L - E_C)^2} = \sqrt{190.8^2 + (449.33 - 303.80)^2}$$

Therefore E = 240 volts.

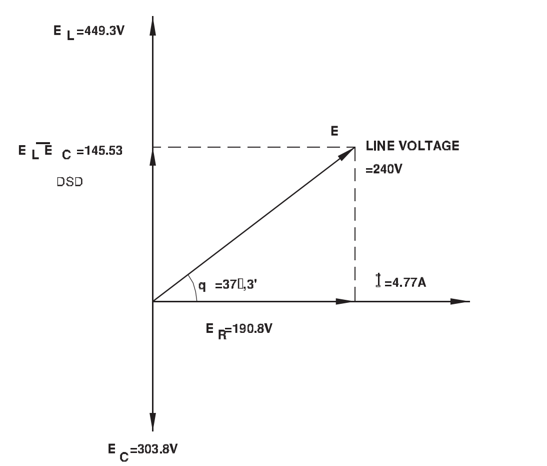
e Vector diagram is shown in Fig 4.

f The calculated voltage and the applied voltages are equal i.e say 240V

g The power factor Cos θ = E_R/ E = 190.8/240 = 0.798.

h The power factor angle is 37°3'.(Refer to Natural cosine table.)

Fig 4



VECTOR DIAGRAM

Application

These AC circuits having R, L and C in series are used in electronic tuning circuits in radio or TV to select the desired station/channel. A variable condenser called gang condenser is used to change the value of X_C equal to X_L at a desired station/channel frequency allowing only resistance in the circuit which, in turn, allows maximum current to flow in the circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

when $X_L = X_C$

Z = R

Current I = V/ R which is maximum.

At this condition the circuit is said to be resonant.

The frequency at resonance

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

as $X_L = X_C$

$2\pi f_R L = 1/2\pi/RC$

Hence

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

AC Parallel circuit problem

In practice all industrial and domestic electrical circuits are connected in parallel as we follow the constant voltage system. In a parallel circuit, the voltage across any branch circuit is the same as the supply voltage. However, the arithmetical sum of the branch currents does not necessarily equal the total current. This is true because the branch current values may be out-of-phase due to the fact that the loads connected may be resistive, inductive, (V lead I) or capacitive (I lead V).

Therefore, the total current must be obtained by adding or subtracting vectors of the branch currents either mathematically (admittance method) or graphically (vector method).

Vector method of solving AC parallel circuit

While drawing vectors for the AC parallel circuit, the following rules need to be followed.

i Draw the line voltage as the horizontal reference line as this voltage is the same across all branch circuits (X axis).

ii Draw the current in the pure resistive branch circuit in phase with the reference vector (X axis) to a scale.

iii Draw the current in the pure inductive branch circuit at 90° lagging the reference vector (Y axis) to the same scale as in I .

iv Draw the current in the pure capacitive branch circuit at 90° leading the reference vector (Y axis) to the same scale as in I.

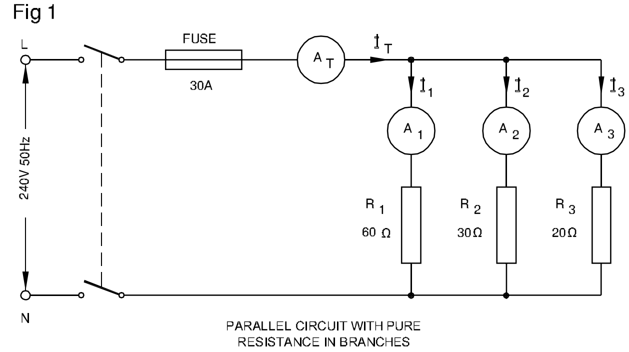
v Use vector subtraction and addition methods to obtain the total current.

Example 1

Parallel circuit with pure resistance

Let us consider an AC parallel circuit having three branches of pure resistance as shown in Fig 1.

Fig 1



Determine the following for the circuit shown in Fig 1.

i The current taken by each branch (I_1, I_2 & I_3).

ii Vector diagram of branch currents and voltage.

iii The line current I_T.

iv The combined resistance.

v The power factor angle and the power factor.

vi The total power taken by the parallel circuit.

solution

i The branch current $I_1 = \frac{V}{R_1}$

$$= \frac{240}{60} = 4 \text{ amps}$$

Pure resistive, hence, in phase with the voltage.

The branch current $I_2 = \frac{V}{R_2}$

$$= \frac{240}{30} = 8 \text{ amps}$$

Pure resistive, hence, in phase with the voltage.

The branch current $I_3 = \frac{V}{R_3}$

$$= \frac{240}{20} = 12 \text{ amps}$$

Pure resistive, hence, in phase with the voltage.

ii Now draw the vector diagram following the rules mentioned above.

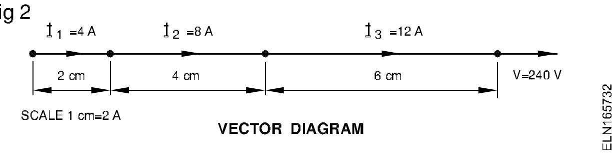
Decide a scale 1cm = 2 amps. (Fig 2)

iii Total current I_T is the sum of the branch currents I_1, I_2 and I_3 as they are in phase with each other.

$I_T = I_1 + I_2 + I_3$

$= 4 + 8 + 12 = 24$ amps.

Fig 2



VECTOR DIAGRAM

iv As all branches have pure resistance load, the total resistance R_T is equal to the total impedance Z.

The total resistance $R_T = Z = \frac{V}{I_T}$

$= \frac{240}{24} = 10$ ohms.

v The power factor angle between the applied voltage and the current is found to be zero as per the vector diagram.

Power factor angle = 0

Power factor = cos ø

= cos 0 = 1 unity.

vi Total power taken by the circuit

$I_T^2 R_T = VI_T \cos ø = 24^2 \times 10$

$= 240 \times 24 = 5760$ watts.

(Total current I_T is in phase with the voltage.)

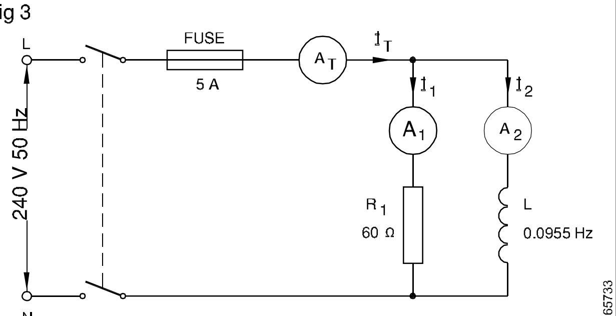
Example 2

Parallel circuit with R and X_L in branches

Now consider a parallel circuit having one branch consisting of a pure resistance and the other branch having pure inductance.

Determine the following for the circuit shown in Fig 3.

Fig 3



AC CIRCUIT R AND L IN PARALLEL

i The branch currents.

ii Draw the vector diagram.

iii The total current.

iv The power factor angle and the power factor.

v The combined impedance.

vi The power in the circuit.

SOLUTION

i The branch current $I_1 = \frac{V}{R_1}$

$= \frac{240}{60} = 4$ amps

Pure resistive, hence, in phase with the voltage.

To calculate the branch current I_2 first find out the inductive reactance X_L.

$X_L = 2\pi FL = 2 \times \frac{22}{7} \times 50 \times 0.0955$

$= 30$ ohms.

So the branch current $I_L = \frac{V}{X_L} = \frac{240}{30} = 8$ amps.

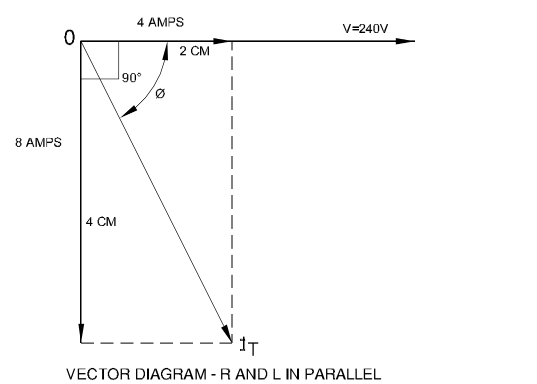
Pure inductive, hence, lags the applied voltage by 90°.

ii Draw the vector diagram by following the rules: Scale 1 cm = 2 amps. (Fig 4)

Complete the parallelogram to find the total current I_T.

Measure the angle ø and the length of $0I_T$.

Fig 4



VECTOR DIAGRAM - R AND L IN PARALLEL

iii Measured angle is 63º 26'

Power factor = Cos 63º 26'

= 0.447 lagging.

iv Length of $0I_T$ = 4.47 cm.

Hence, I_T = 4.47 x 2 = 8.94 amps.

The combined impedance of the circuit = Z.

v Power taken by the circuit

$P = VI \cos ø = I_1^2R$

= 240 x 8.94 x 0.447 = 4^2 x 60

= 959 watts approx. 960 watts.

Example 3

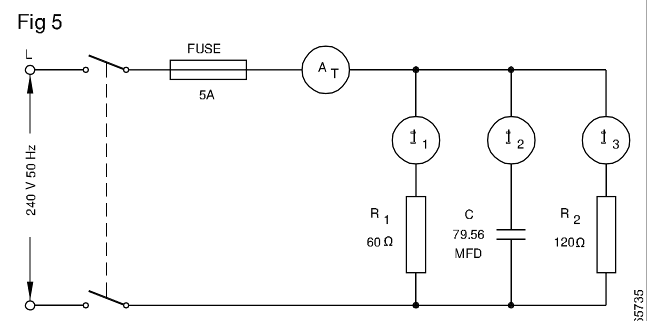
Parallel circuit with R and X_C

Now consider a parallel circuit having pure resistance in two branches and a pure capacitance in the third branch.

Find the following for the circuit shown in Fig 5.

i The branch currents.

ii Vector diagram of the branch currents.



AC CIRCUIT - R, L AND C IN PARALLEL

iii Total current I_T.

iv Power factor angle.

v Power factor.

vi Power in the circuit.

Solution

i The branch current $I_1 = \frac{V}{RI} = \frac{240}{60}$ = 4 amps

Pure resistive, hence, in phase with the voltage.

To calculate the branch current I2 first find out the capacitive reactance X_C.

$$X_C = \frac{1}{2\pi FC} = \frac{1}{2 \times 3.142 \times 50 \times 79.56 \times 10^{-6}} = 40\Omega$$

So the branch current $I_2 = \frac{V}{X_C} = \frac{240}{40}$ = 6 amps

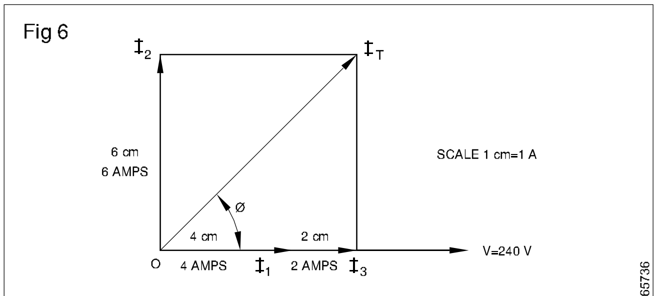
Pure capacitive, hence, current leads the applied voltages by 90º.

The branch current $I_3 = \frac{V}{R} = \frac{240}{120}$ = 2 amps

ii Draw the vector diagram to scale.

Complete the parallelogram to find the total current I_T. (Fig 6)

iii Measured length OI_T = 8.5cm.



VECTOR DIAGRAM - R, L AND C IN PARALLEL

Total current I_T. = 8.5 x 1 = 8.5 amps.

iv Measure the angle between the total current and the voltage.

Measured angle θ = 45º leading.

v Power factor cos ø = cos 45º = 0.707.

vi Power taken by the circuit.

$P = VI \cos \theta = (I_1^2R_1 + I_3^2R_2)$ = 240 x 85 x 0.707

= (4^2 x 60 + 2^2 x 120)

1442 approx.1440 watts.

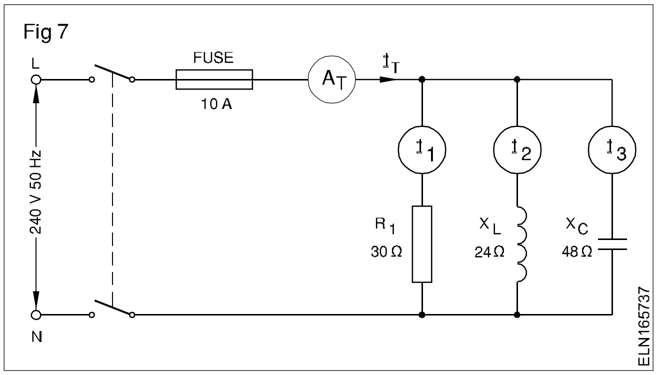
Example 4

Parallel circuit with R, X_L and X_C

Now consider a parallel circuit having pure resistance in one branch, pure inductance in the 2nd branch and a pure capacitance in the 3rd branch as shown in Fig 7.

Find the following for the circuit shown in Fig 7.

i The branch currents.



ii The vector diagram.

iii Total current I_T.

iv Power factor angle.

v Power factor.

vi Power taken by the circuit.

vii Impedance of the circuit.

SOLUTION

i The branch current

$$I_1 = \frac{V}{R_1} = \frac{240}{30} = 8 \text{ amps in phase with V.}$$

The branch current

$$I_2 = \frac{V}{X_L} = \frac{240}{24} = 10 \text{ amps lagging 'V' by } 90^o.$$

The branch current

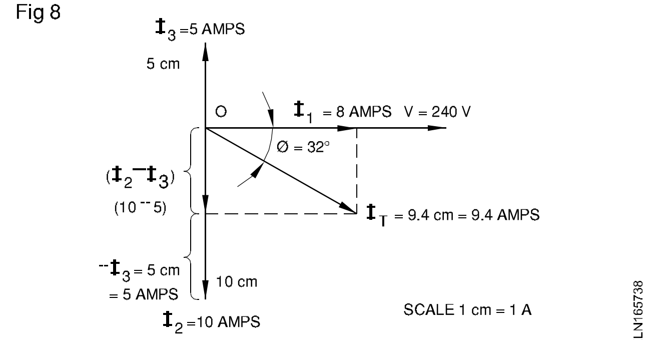
$$I_3 = \frac{V}{X_C} = \frac{240}{48} = 5 \text{ amps leading 'V' by } 90^o.$$

ii Draw the vector diagram to scale.

Scale 1 cm = 1 ampere

Complete the parallelogram to find the total current I_T (Fig 8).

Fig 8



iii Measured OI_T = 9.4 cm.

Total current

I_T = 9.4 x 1 = 9.4 amps.

iv Measure the angle between voltage and the total current.

Measured angle = 32º lagging.

v Power factor cos θ = cos 32º = 0.85.

vi Power taken by the circuit

$VI \cos\theta = I_1^2 R$

$= 240 \times 9.4 \times 0.85 = 8^2 \times 30$

= 1918 approx.1920 watts.

vii Combined impedance Z

$$Z = \frac{V}{I_T} = \frac{240}{9.4} = 25.5 \text{ ohms}$$

Admittance method of solving AC parallel circuit

In solving problems in AC circuit of parallel groups either the vector or the admittance method could be used. However, there will be considerable difficulty in solving problems by vector method if series parallel combination groups are to be dealt with.

Though admittance method requires simple knowledge of mathematics, the numbers to be handled are decimals, their addition, subtraction, square and roots will make the solutions a little more cumbersome.

Let us find how this method could be used to solve problems in parallel AC circuits.

When several impedances say Z_1, Z_2 and Z_3 are connected in parallel, their combined impedance Z could be found by

$$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \text{(Equation .... 1)}$$

$$\frac{1}{Z} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \text{...............} + \frac{1}{Z_n} \text{ (Vector sum)}$$

Alternatively

$$Y = Y_1 + Y_2 + Y_3 \text{ Where } \frac{1}{Z} = Y$$

where the reciprocal impedance is called admittance, the unit is Siemens and the symbol is Y.

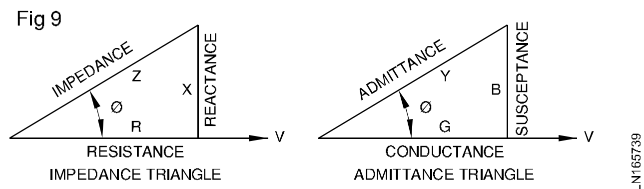
Just like impedance, the admittance also has two components as shown in Fig 9.

One component which is in phase with the voltage is called conductance, the unit is Siemens, and the symbol is G.

The other component which is in quadrature with the applied voltage V is called susceptance, the unit is Siemens and the symbol is B.

Admittance $Y = Y_1 + Y_2 + Y_3$ vectorially.

Fig 9



From the admittance triangle we have

$$Y = \sqrt{G^2 + B^2} \text{ .... Eqn.}$$

$$G = Y \cos\theta \text{ .... Eqn.}$$

Where $Y = \frac{1}{Z}$ and $\sin\theta = \frac{R}{Z}$

Hence $G = Y \times \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$ .... Eqn.

$B = Y \sin\theta$ .... Eqn.

Where $Y = \frac{1}{Z}$ and $\sin\theta = \frac{X}{Z}$

Hence $B = \frac{1}{Z} \times \frac{X}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$ .... Eqn.

Further when several resistances, reactances are connected in parallel the conductances of individual branches can be added to get the total conductance

$G = G_1 + G_2 + G_3 + ...+G_m$

Likewise when several reactances are connected in parallel, the susceptance of individual branches can be added algebraically to get the total susceptance. The susceptance due to inductive reactances are taken as +ve sign where the susceptance due to capacitive reactances are taken as –ve sign.

$B = b_1 + b_2 + (-b_3)$ ....

Example 1

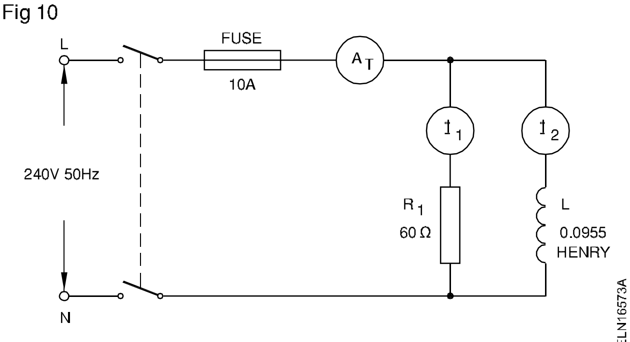
Parallel circuit with R and XL in branches.

Determine the following for the circuit shown in Fig 10.

i Conductance in branch circuits:

The conductance $G = g_1 + g_2$

Fig 10



where g_1 and g_2 are the conductance of branch 1 and 2 respectively.

In branch 1

$$g_1 = \frac{R_1}{R_1^2 + X_1^2} = \frac{60}{60^2 + 0^2}$$

$$= \frac{60}{60^2} = \frac{1}{60} = 0.01667 \text{ Siemens}$$

$$b_1 = \frac{X}{R_1^2 + X_1^2} = \frac{0}{60^2 + 0^2}$$

In branch 2

$$X_L = 2\pi fL = 2 \infty \frac{22}{7} \infty 50 \infty 0.0955 = 30 \text{ ohms}$$

$$g_2 = \frac{R_1}{R_2^2 + X_2^2} = \frac{0}{0^2 + 30^2} = 0$$

$$b_2 = \frac{X}{R_L^2 + X^2} = \frac{30}{0^2 + 30^2} = \frac{1}{30} = 0.033 \text{Siemens}$$

Admittance $Y = \sqrt{G^2 + B^2}$

where $G = g_1 + g_2 = 0.01667 + 0 = 0.01667$ Siemens

and $B = b_1 + b_2 = 0 + 0.0333 = 0.0333$ Siemens

i.e $Y = \sqrt{0.01667^2 + 0.0333^2} = 0.0372$ Siemens

= 0.0372 Siemens

The branch current $I_1 = \frac{V}{Z_1}$

$\frac{V}{R} = \frac{240}{60} = 4$ amps in phase with the voltage

The branch current $I_2 = \frac{V}{Z_2}$

$\frac{V}{X_L} = \frac{240}{30} = 8$ amps

lagging behind the applied voltage by 90°.

Total current = $I_T = \sqrt{I_1^2 + I_2^2}$

$$= \sqrt{4^2 + 8^2} = \sqrt{16 + 64}$$

= 8.94 amps

Alternatively, $I = \frac{V}{Z}$ = VY = 240 x 0.0372

= 8.94 amps.

Power factor $= \frac{G}{Y} = \frac{I_1}{I_T}$

$= \frac{0.01667}{0.0372} = \frac{4}{8.94}$ = 0.448 approx. 0.447.

So power factor angle = 63° 26’.

Impedance of the circuit $Z = \frac{1}{Y} = \frac{1}{0.0372} = 26.88$ ohms

Power taken by the circuit = VI cos ø

= 240 x 8.94 x 0.447

= 959 watts.

Example 2

In Fig 11, Parallel circuit with R, X_L and X_c

Find the following.

i Conductance and susceptance of each branch.

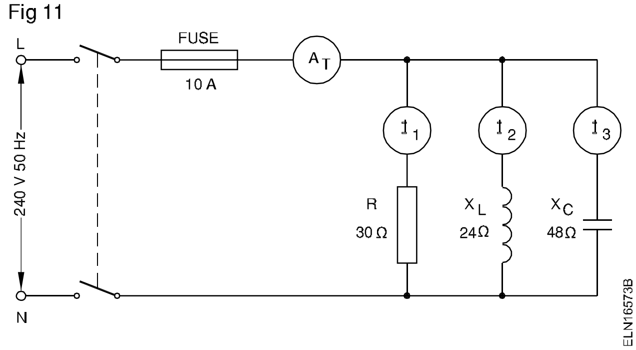
ii Total G, B and Y.

iii Branch currents.

iv PF and PF angle.

v Power taken by the circuit.

Fig 11



i Conductance in branch circuits

$$g_1 = \frac{R_1}{Z_1^2} = \frac{30}{30^2} = \frac{1}{30}$$

= 0.0333 siemens

$$g_2 = \frac{R_2}{Z_2^2} = \frac{0}{24^2} = 0$$

$$g_3 = \frac{R_3}{Z_3^2} = \frac{0}{48^2} = 0$$

Susceptance in branch cirucits

$$b_1 = \frac{X_1}{Z_1^2} = \frac{0}{30^2} = 0$$

$$b_2 = \frac{X_2}{Z_2^2} = \frac{24}{24^2} = \frac{1}{24}$$

= 0.04167 siemens

$$b_3 = \frac{-X_3}{Z_1^2} = \frac{-48}{-48^2} = -\frac{1}{48}$$

= - 0.02083 siemens

ii Total conductance G = $g_1 + g_2 + g_3$

= 0.0333 + 0 + 0

= 0.0333 Siemens.

Total susceptance B = $b_1 + b_2 + b_3$

= 0 + 0.04167 + (– 0.02083)

= 0.02084 Siemens.

$$Y = \sqrt{G^2 + B^2}$$

$$= \sqrt{0.333^2 + 0.02084^2}$$

= 0.03928 Siemens.

iii The branch current $I_1 = \frac{V}{Z_1}$

$$= \frac{V}{R} = \frac{240}{30}$$ = 8 amps in phase with V

The branch current $I_2 = \frac{V}{Z_2}$

$$\frac{V}{X_L} = \frac{240}{24} = 10$$ amps lagging 90° with V

The branch current $I_3 = \frac{V}{X_3}$

$$= \frac{240}{48}$$ = 5 amps lagging 90° with V

Total current

$$I_T = \sqrt{I_1^2 + (I_2 - I_3)^2}$$

$$= \sqrt{8^2 + (10-5)^2} = \sqrt{89}$$

= 9.43 amps

Alternatively

I_T = VY = 240 X 0.03928

= 9.43 amps.

iv Power factor $= \frac{G}{Y} = \frac{I_R}{I_T}$

$= \frac{0.0333}{0.03929} = \frac{8}{9.43}$

= 0.848.

v Power factor angle = 32° lagging.

Power taken by the circuit = VIcos ø

= 240 x 9.43 x 0.848

= 1919 watts.

Total impedance = Z = $\frac{1}{Y}$

$\frac{1}{0.03929}$ = 25.5.ohms

Check these answers with the answers obtained by the vector method.

Power factor - improvement of power factor

Objectives: At the end of this lesson you shall be able to
- **define power factor - explain the causes of low power factor**
- **list out disadvantage of low power factor and advantage of higher power factor in a circuit**
- **explain the methods to improve the power factor in an AC circuit**
- **illustrate the importance of power factor improvement in industries**
- **distinguish between leading, lagging and zero PF**
- **state the recommended power factor as per ISI 7752 (Part I) 1975 for electrical equipment.**

Power Factor (P.F.)

The power factor is defined as the ratio of true power to apparent power and it is denoted by Cos θ.

$$\text{i. e. Power Factor} = \frac{\text{True Power } (W_T)}{\text{Apparent Power } (W_a)} = \text{Cos } \theta$$

$$\text{or Cos } \theta = \frac{W_T}{V \times I}$$

Where W_T is the real power (true power) and is measured in watts or some times in kilowatts (kW). Similarly the product VI is known as the apparent power measured in volt amperes or sometimes in kilo-volt amperes written as kVA.

The majority of AC electrical machines and equipment draw from the supply the apparent power (kVA) which exceeds the required useful power (KW). This is due to the reactive power (kVAR) necessary to produce the alternating magnetic field in motors and transformers.

The ratio of useful power (kW) to apparent power (kVA) is termed the PF of the load. The reactive power is indispensable and constitutes an additional demand on the system.

The principal cause of a low power factor is due to the reactive power flowing in the circuit. The reactive power mostly due to inductive load rather than capacitive load.

Variation in power factor and the type of circuits

The following are the different conditions of the power factor in different circuits.

Unity power factor

A circuit with a unity power factor will have equal real and apparent power, so that the current remains in phase with the voltage, and hence, some useful work can be done. (Fig 1a)

Leading power factor

A circuit will have a leading power factor if the current leads voltage by an angle q of electrical degrees and the true power will be less than the apparent power. Mostly capacitive circuits and synchronous motors operated at over excitation contribute for leading power factor. (Fig 1b)

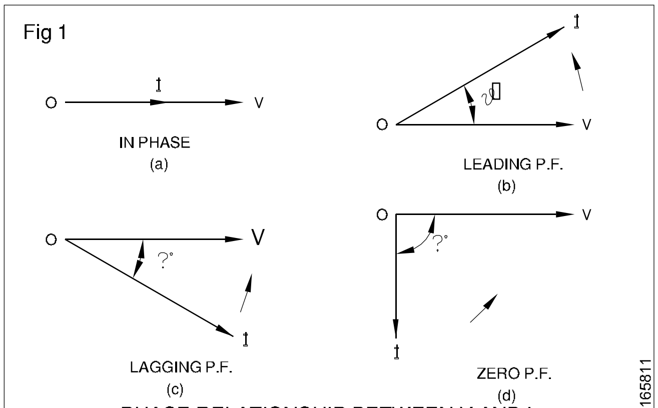
Lagging power factor

In such a circuit the true power is less than the apparent power and current lags behind the voltage by an angle, in electrical degrees. Mostly inductive loads like induction motors and induction furnaces account for lagging power factor. (Fig 1c)

Zero power factor

When there is a phase difference of 90° between the current and voltage, the circuit will have zero power factor and no useful work can be done. Pure inductive or pure capacitive circuits account for zero power factor. (Fig 1d)

Fig 1



PHASE RELATIONSHIP BETWEEN V AND I

The power factor can be one or less than one but can never be greater than one.

Table 1 shows the most common electrical appliances used, the power in watts and the average power factor.

TABLE 2 shows the natural power factor of the various installations used in the industries.

TABLE 1

Power factor for single phase electrical appliances and equipment (Reference IS 7752 (Part I) - 1975)

Sl.No.	Appliance/Equipment	Power output		Average natural power factor
		Min.(W)	Max.(W)	
1	Neon sign	500	5000	0.5 to 0.55
2	Window type air- conditioners	750	2000*	0.75 to 0.85 0.68 to 0.82 0.62 to 0.65
3	Mixer	150	450	0.8
4	Coffee grinder	200	400	0.75
5	Refrigerator	200	800	0.65
6	Freezer	600	1000	0.7
7	Shaver	80	250	0.6
8	Table fan	25	120	0.5 to 0.6
9	Ceiling fan	60	100	0.5 to 0.7
10	Exhaust fan	150	350	0.6 to 0.7
11	Sewing machine	80	120	0.7 to 0.8
12	Washing machine	300	450	0.6 to 0.7
13	Radio	25	450	0.8
14	Vacuum cleaner	200	450	0.7
15	Tube light	40	100	0.5
16	Clock	5	10	0.9

* Starts dropping when compressor motor is not in circuit.

TABLE 2

Power factor for three-phase electrical installations (Reference IS 7752 (Part I) - 1975)

Sl.No.	Type of installation	Natural power factor
1	Cold storage and fisheries	0.7 to 0.80
2	Cinemas	0.78 to 0.80
3	Confectionery	0.77
4	Dyeing and printing (Textile)	0.60 to 0.87
5	Plastic moulding	0.57 to 0.73
6	Film studios	0.65 to 0.74
7	Heavy engineering works	0.48 to 0.75
8	Pharmaceuticals	0.75 to 0.86
9	Oil and paint manufacturing	0.51 to 0.69
10	Printing press	0.65 to 0.75
11	Food products	0.63
12	Laundries	0.92
13	Flour mill	0.61
14	Textile mills	0.86
15	Oil mill	0.51 to 0.59
16	Woolen mills	0.70
17	Cotton press	0.63 to 0.68
18	Foundries	0.59
19	Tiles and mosaic	0.61
20	Chemicals	0.72 to 0.87
21	Rolling mills	0.72 to 0.60
22	Irrigation pumps	0.50 to 0.70

Causes of low power factor

The following are the reasons.

i In industrial and domestic fields, the induction motors are widely used. The induction motors always take lagging current which results in low power factor.

ii The industrial induction furnaces have low power factor.

iii The transformers at substations have lagging power factor because of inductive load and magnetising currents.

iv Inductive load in houses like fluorescent tubes, mixers, fans etc.

The disadvantages of low power factor are as follows.

a For a given true power, a low power factor causes increased current, thereby, overloading of the cables, generators, transmission and distribution lines and transformers.

b Decreased line voltage at the point of application (voltage drop at consumer end) due to voltage drop and power losses in the supply system.

c Inefficient operation of plant and machine (efficiency drops due to low voltage).

d Penal power rates (increased electricity bills).

The advantages of high power factor are as follows.

As the higher PF for a given load, reduces the current, there will be:

a a possibility of connecting extra load on existing generators and transmit additional power through the same lines

b lesser losses and voltage drop in lines; thereby, transmission efficiency is high and the voltage at the point of application will be normal without much drop

c normal voltage improves the efficiency of operation of plants and machinery

d reduction in electricity bills for the given load during the given time.

Method of improving the power factor

To improve the power factor of a circuit, two methods are used:

i to run a lightly loaded synchronous motor with over-excitation on that line in which the PF is to be improved

ii to connect capacitors in parallel with the load.

Usually the capacitor method is used in Indian factories.

Synchronous condenser method

The synchronous motor is used in certain industries as well as in receiving end substations to drive a mechanical load and also to correct the power factor. An over-excited synchronous motor draws leading current to compensate the lagging current taken by the other loads.

The leading volt-ampere reactive power taken by a synchronous motor, when over- excited will be opposite in nature to the lagging voltage pure reactive due to inductive loads, and, thereby, reduces the volt-ampere reactive component to improve the power factor.

Example

A factory is having a load of 100 kW working at 0.6 PF lagging. A synchronous motor is connected in the factory and is made to run over-excited to improve the power factor. The synchronous motor is of 30 kW and is working at 0.8 PF leading. Calculate the following:

i the true power in watt, aspperent power in VAR for the factory load at 0.6p.f lagging.

ii The true power in watt, apparent power in volt- ampere and leading reactive power in VAR for the synchoronous motor at 0.8P.F lagging.

iii The true power in watt, reactive power in VAR and apparent power in Volt - ampere and PF supplied by the feeder lines.

i Factory Load

Load in kW = 100 kW

Load in watts = 100×10^3 watts

Load in volt-amperes = $\frac{\text{True power}}{\text{PF}} = \frac{100\text{x}10^3}{0.6}$

= 167×10^3 volt - amperes

Load in vars = Volt ampere x sin θ

= Cos θ = 0.6

= θ = 53.1°

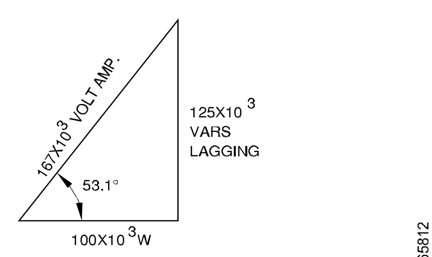
= Sin θ = Sin 53.10 = 0.8

Load in vars = $167 \times 10^3 \times 0.8$

= 133.6×10^3 vars lagging

Refer to Fig 2.

Fig 2



FACTORY LOAD - VECTOR REPRESENTATION

ii Synchronous motor

Motor load in kW = 30 kW = 30×10^3 watts

Motor load in volt - amperes= $\frac{\text{True power}}{\text{PF}} = \frac{30\text{x}10^3}{0.8}$

= 37.5×10^3 volt - amperes

Motor load in vars = Volt ampere sin θ

Cos θ = 0.8

θ = 36.1°

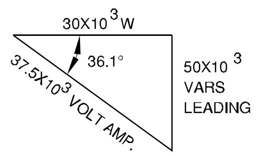
Sin θ = Sin 36.1° = 0.6

Motor load in vars = $37.5 \times 10^3 \times 0.6$

= 22.5×10^3 vars lagging

Refer to Fig 3.

Fig 3



SYNCHRONOUS MOTOR - VECTOR REPRESENTATION

iii Feeder line

Condition: Combined load condition for improvement of PF by the synchronous motor

Total load in watts = Factory true power + true power taken by synchronous motor

= $100 \times 10^3 + 30 \times 10^3$

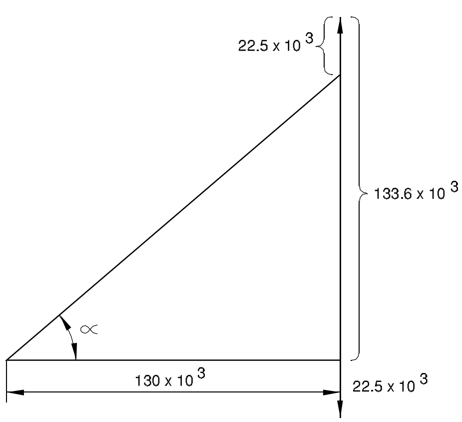
= 130×10^3 watts

Total load in V_{ARS} = factory reactive power (Inductive) } syn. motor reactive power (Capacitance)

= $133.6 \times 10^3 - 22.5 \times 10^3$ V_{ARS} lagging

= 111.1×10^3 V_{ARS} lagging

Fig 4



LOAD + SYNCHRONOUS MOTOR
(VECTOR REPRESENTATION)

This could be represented by a vector diagram as shown in the Fig 4.

$$\text{Now Tan } \alpha = \frac{\text{Opp. side}}{\text{Adj. side}} = \frac{111.1\text{x}10^3}{130\text{x}10^3} = 0.8546$$

Angle α = 40.5°

Power factor of the factory after the connection of synchronous motor = Cos θ = Cos 40.5° = 0.7604

The PF has improved from 0.6 to 0.7604 by the use of the synchronous motor.

Present volt-amperes supplied by the factory

$$= \frac{\text{True power}}{\text{PF}} = \frac{\text{Truepower}}{\text{Cos}\,\alpha}$$

$$= \frac{130 \times 10^3}{\text{Cos } 40.5^\circ} = \frac{130 \times 10^3}{0.7604}$$

= 171×10^3 Volt amperes

Condenser method

Capacitors when used for PF improvement are connected in parallel to the supply. In three-phase circuits the capacitors are connected in delta across the load lines. Now automatic devices are available which can be connected to the supply lines to detect low power factor and to switch on the required capacity of capacitors in the line to improve the power factor.

Normally these capacitors are provided with discharge resistances to discharge the stored energy. However, no capacitor terminal should be touched to avoid shock.