Mensuration - Area and perimeter of square, rectangle and parallelogram

Exercise 1.8.39

In Engineering field, an Engineer has to estimate the material, manpower, machinery, etc. required to prepare the geometrical objects. Hence we must be very conversant with all relevant formulae connected with geometrical objects.

Length - l unit

Breadth or width - b unit

Diagonal - d unit

Diameter - d unit

Radius - r unit

Semiperimeter - S unit

Perimeter - P unit

Circumference - C unit

Area - A unit^2

Total surface area - T.S.A unit^2

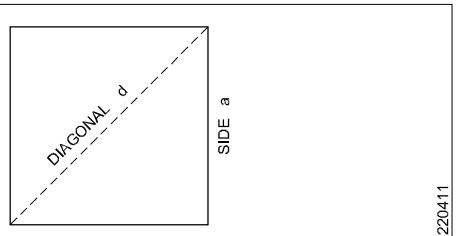
Lateral surface area - L.S.A unit^2

Volume - V unit^3

Square

This is also a four sided figure, opposite sides are parallel. All the four sides are equal. All the sides are inclined at 90º.

Fig 1



$A = a^2$ (or) unit^2

$P = 4a$ unit

$d = \sqrt{2}\ a$ unit

$a = \frac{d}{\sqrt{2}}$ unit where $\sqrt{2} = 1.414$

Find the area of a brass sheet in the form of a square whose perimeter is 31.2 cm.

Perimeter(P) = 4a = 31.2 cm

$\therefore a = \frac{31.2}{4} = 7.8$ cm

Area (A) $= a^2$

$= 7.8 \times 7.8 = 60.84$ cm^2

Examples

1 Find out the circumference, diagonal and area of a square, whose side is 18 cm.

Side of the square (a) = 18 cm

Perimeter (P) = 4a

= 4 x 18 = 72 cm

Diagonal (d) = $\sqrt{2}$ x a

= $\sqrt{2}$ x 18 = 1.414 x 18

= 25.45 cm

Area (A) = a^2

= 18 x 18 = 324 cm^2

Perimeter of square = 72 cm

Diagonal = 25.45 cm ; Area = 324 cm^2

2. If the diagonal of a square measure 10 cm. Find the area of the square.

Diagonal of the square (d) = $\sqrt{2}a$ = 10 cm

Side (a) = $\frac{d}{\sqrt{2}}$

Area (a^2) = $\frac{d^2}{2}$

$= \frac{10^2}{2} = \frac{100}{2}$

= 50 cm^2

Area of the square = 50 cm^2

3. The perimeter of one square is 748 cm and that of another is 336 cm. Find the perimeter of a square which is equal in area of the sum of the two.

Side of the square (a) = $\frac{\text{Perimeter}}{4}$

1st square

Side (a) = $\frac{\text{Perimeter of I}^{st}\text{ square}}{4}$

$= \frac{748}{4} = 187$ cm

Area (A) = a^2

= 187 x 187

= 34,969 cm^2

2nd square

Side (a) = $\frac{\text{Perimeter of II}^{nd}\text{ square}}{4}$

$= \frac{336}{4} = 84$ cm

Area (A) $= a^2$

$= 84 \times 84$

$= 7{,}056\ cm^2$

Total area of two squares = 34,969 + 7,056

$= 42{,}025\ cm^2$

Total area of two squares = 3rd square area

3rd square area $= a^2 = 42{,}025\ cm^2$

Side (a) $= \sqrt{42{,}025} = 205$ cm

Perimeter (P) $= 4 \times a$

$= 4 \times 205$

$= 820$ cm

Perimeter of 3rd square = 820 cm

Assignment

1 Find the Area, Perimeter and diagonal of a square steel plate whose side measures 28.1 cm.

2 Find the area of a square whose diagonal is equal to 8.5 cm.

3 Find the area of the square if the side of the square is 28 cm.

4 Find its side if the area of the square field is 169 m^2.

5 Find the area of the square if the diagonal of the square is 20 cm.

6 Find the perimeter of a square whose diagonal is 144 m.

7 Find the area if the perimeter of a square plot is 48 m.

Rectangle

This is a four sided figure. Opposite sides are parallel. Angles between adjecent sides are 90°.

A = Area = length x breadth = l.b.unit^2

P = Perimeter = 2 (l + b) unit

Diagonal = $\sqrt{l^2 + b^2}$ unit

Examples

1 Find the Area, Perimeter and diagonal of a rectangle whose length and breadth are 144 mm and 60 mm respectively.

Area = A $= l \times b$ unit^2

$= 144 \times 60 = 8640\ mm^2$

Perimeter = P $= 2 (l + b)$ unit

$= 2(144 + 60)$

$= 2 \times 204 = 408$ mm

Diagonal = d $= \sqrt{l^2 + b^2}$ unit

$= \sqrt{144^2 + 60^2}$

$= \sqrt{20736 + 3600}$

$= \sqrt{24336} = 156$ mm

2 The perimeter of a rectangle is equal to 42 cm. If its breadth is 9 cm. Find the length of the rectangle.

P = 42 cm

b = 9 cm

l = ?

$P = 2(l + b) = 42$

$2(l + 9) = 42$

$l + 9 = 42 \div 2$

$l + 9 = 21$

$l = 21 - 9$

$l = 12$ cm

3 The perimeter of a rectangle is 48 cm and its length is 4 cm more than its width. Find the length and breadth of the rectangle.

P = 48 cm

b = x

l = $x + 4$

$2(l + b) =$ Perimeter

$2(x + 4 + x) = 48$

$2(2x + 4) = 48$

$4x + 8 = 48$

$4x = 48 - 8$

$x = \frac{40}{4} = 10$

x = breadth = 10 cm

length $= x + 4 = 10 + 4$ **= 14 cm**

4 How many rectangular pieces of 50 cm x 20 cm can be cut out from a sheet of 1000 cm x 500 cm.

Sheet size = 1000 cm x 500 cm

Size of the rectangular piece to be cut = 50 cm x 20 cm

No. of pieces to be cut in lengthwise = $\frac{1000}{50}$ = 20

No. of pieces to be cut in breadthwise = $\frac{500}{20}$ = 25

Total no. of pieces to be cut out = 20 x 25

= 500

5. The perimeter of a rectangle is 320 metre. Its sides are in the ratio of 5:3. Find the area of the rectangle.

Ratio = 5:3 = l : b

length l = $5x$

breadth b = $3x$

2(l + b) = Perimeter

$2(5x + 3x)$ = 320

$2(8x)$ = 320

$16x$ = 320

$x = \frac{320}{16} = 20$

l = $5x$ = 5 x 20 = 100 m

b = $3x$ = 3 x 20 = 60 m

Area = l x b

= 100 x 60

= 6000 m^2

Assignment

1 Find the area of a rectangular plot whose sides are 24 metres and 20 metres respectively. Also find the perimeter of the plot.

2 How many rectangular pieces of 5 cm x 4 cm will you get out of 65 cm x 30 cm brass sheet?

3 Find its breadth and area if the perimeter of a rectangle is 400 metre and its length is 140 m. .

4 Find its area, if the opposite sides of a rectangle are 64 cm and 25 cm respectively.

5 What is the width of the rectangle if a rectangle has an area of 224 cm^2 and length 16 cm.

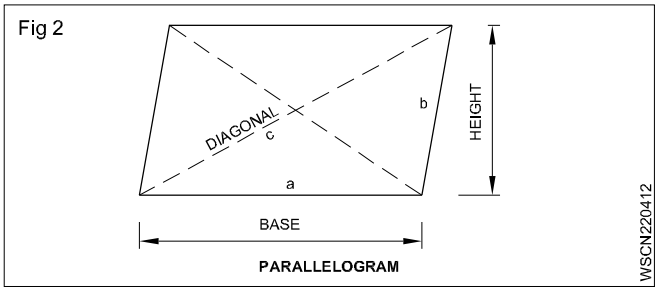
6 What is the length of the diagonal of a rectangle with sides 16 cm and 12 cm?

7 Find the area of the rectangle if the perimeter of the rectangle is 100 cm and the ratio of its length and breadth is 3:2.

Parallelogram

This is also a four sided figure, opposite side being parallel to each other.

Fig 2



Area of parallelogram = base x height

or = $2x\sqrt{s(s-a)(s-b)(s-c)}$

Where

$s = \frac{a+b+c}{2}$

a,b and c are adjacent sides.

P = 2(a+b)

Examples

1 The base and height of a paralleogram are 7.1 cm and 2.85 cm. Calculate its area.

A = base x height

= 7.1 x 2.85

= 20.235 cm^2

2 Find the height of a parallelogram whose area is 20 cm^2 and base is 10 cm.

A = base x height

$h = \frac{area}{base}$

$= \frac{20}{10}$

= 2 cm

3. Two sides of a parallelogram are 12 cm and 8 cm. The diagonal is 10 cm long. Find the area of the parallelogram.

$$A = 2x\sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

$$= \frac{12+8+10}{2}$$

$$= \frac{30}{2}$$

$$= 15$$

$$A = 2x\sqrt{15(15-12)(15-8)(15-10)}$$

$$= 2x\sqrt{15 \times 3 \times 7 \times 5}$$

$$= 2x\sqrt{1575}$$

$$= 2 \times 39.686$$

$$= 79.37 \text{ cm}^2$$

Assignment

1 Find the area of a parallelogram, if its base and height are 8.1 cm and 30.8 cm respectively.

2 Find the area of a parallelogram, if the sides of a field in the shape of parallelogram are 12 m and 17 m and one of the diagonal is 25 m.

3 Find the base of a parallelogram whose height is 12 cm and area is 120 cm^2.

4 Find the height of a parallelogram whose base is 40 cm and area is 320 cm^2.

5 Find the area of the land if the sides of a land in the shape of a parallelogram are 24 m and 28 m respectively and one of the diagonal is 30 m.

6 What is the perimeter of parallelogram if base is 10 cm and other side is 5 cm?

7 Find the area of parallelogram if its base and height are 25 cm and 12 cm.

8 Find the base of a parallelogram if height is 15 cm and area is 150 cm^2.

9 Find the area of parallelogram if side is 5 cm, diagonal is 8 cm and diagonal bisects each other at right angles.

10 Find the height of a parallelogram if base is 80 cm and area is 640 cm^2.

11 Find the area of parallelogram if its base and height are 15 cm and 8 cm.

12 Calculate the perimeter and area of parallelogram if base, height are 12.7 cm, 5.5 cm and other side is 6.5 cm

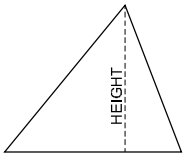
13 Find the height of parallelogram if the area is 20 cm^2 and base is 10 cm

Triangles

Tri means three. Hence tri- angle means three angled figure. For construction of three angled figure, there should be three sides. Hence triangle means three sided figure. Sum of the three angles of any triangle = 180˚.

i Any triangle.

Fig 1



TRIANGLE

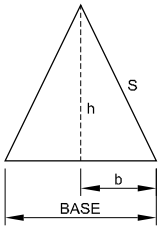
WSCN220511

Area of any triangle = $\frac{1}{2}$ x Base x Height unit^2

ii Isosceles Triangle

In this triangle two of its sides are equal.

Fig 2



ISOSCELES TRIANGLE

WSCN220512

Area of isosceles triangle = $\frac{1}{2}$ x Base x Height

Where

base = 2.b

s = One of equal sides (or) Slant height

h = Height = $\sqrt{s^2 - b^2}$

Area of isosceles triangle = $\frac{1}{2}$ x 2b x $\sqrt{s^2 - b^2}$

= b . $\sqrt{s^2 - b^2}$ unit^2

(Where b= half of base)

(or) Area of Isosceles triangle = $\frac{1}{4}b\sqrt{4a^2 - b^2}$ unit^2

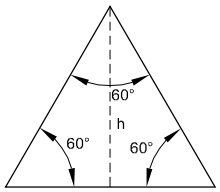
a = Equal sides

b = Base

iii Equilateral triangle

In this triangle all the three sides are equal. Hence angle between adjacent sides is 60˚ (because no. of angles) total = 180˚ ; angle between sides = $\frac{180}{3}$ = 60˚

Fig 3



EQUILATERAL TRIANGLE

WSCN220513

Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ x side^2

= $\frac{\sqrt{3}}{4}$ x a^2 unit^2

Where $\sqrt{3}$ = 1.732

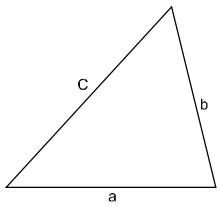
P = 3a unit

P = $\frac{\sqrt{3}}{2}$a unit

iv Scalene triangle

In this triangle the sides are not equal. Angles between the sides, are also not equal. we may also call this triangle as irregular triangle.

Fig 4



SCALENE TRIANGLE

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Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ unit^2

where

a,b,c are sides of triangle

s = Semi perimeter = $\frac{a+b+c}{2}$ unit

v Right angled triangle

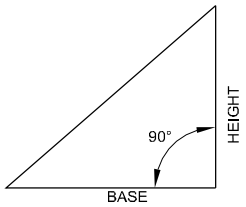
In this triangle, angle between one of two adjacent sides is 90^0. Right angle means ninety degrees. That's why right angled triangle means, one of the angles of this triangle is definitely ninety degrees.

Area of right angled triangle

= $\frac{1}{2}$ x Base x Height

= $\frac{1}{2}$ bh unit^2

Fig 5



RIGHT ANGLED TRIANGLE

WSCN220515

Hypotenuse $= \sqrt{Base^2 + Height^2}$

Where hypotenuse means, the diagonal or largest length of the side of right angled triangle.

Examples

1 Calculate its area if the base and height of a triangle are 10 cm and 3.5 cm respectively.

Base (b) = 10 cm

Height (h) = 3.5 cm

Area (A) = ?

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 10 \times 3.5$

= 17.5 cm²

2 Calculate the base of a triangle having an area of 15 cm² and height is 3.5 cm.

Area (A) = 15 cm²

Height (h) = 3.5 cm

Base (b) = ?

$\frac{1}{2} \times b \times h = A$

$\frac{1}{2} \times b \times 3.5 = 15$

$b = \frac{15 \times 2}{3.5}$

= 8.57 cm

3 Calculate the height of a triangle whose area is 60 cm² and base is 10 cm.

Area (A) = 60 cm²

Base (B) = 10 cm

Height (h) = ?

$\frac{1}{2} \times b \times h = A$

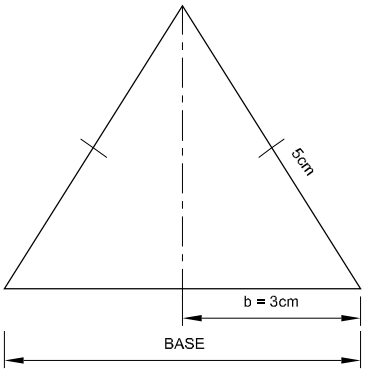
$\frac{1}{2} \times 10 \times h = 60$

$h = \frac{60 \times 2}{10}$

height h = 12 cm

4 Find the area of an isosceles triangle whose base is 6 cm long and each of the other two sides 5 cm long.

Fig 6



ISOSCELES TRIANGLE

WSCN220516

Base (b) = 6 cm = $\frac{6}{2}$ = 3 cm

Equal sides or slant height 's' = 5 cm

Area (A) = ?

$A = b \times \sqrt{s^2 - b^2}$

$= 3 \times \sqrt{5^2 - 3^2}$

$= 3 \times \sqrt{25 - 9}$

$= 3 \times \sqrt{16}$

$= 3 \times 4$

$= 12 \text{ cm}^2$

or

$A = \frac{1}{4} b \sqrt{4a^2 - b^2}$

$= \frac{1}{4} \times 6 \sqrt{4 \times 5^2 - 6^2}$

$= \frac{1}{4} \times 6 \times 8$

= 12 cm²

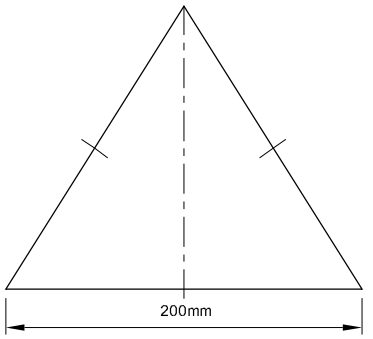
5 Find its height if an isosceles triangle has base of 200 mm and its area is 2000 mm².

Base = 200 mm

Area = 2000 mm²

h = ?

Fig 7



ISOSCELES TRIANGLE

WSCN220517

$\frac{1}{2}$ x b x h = A

$\frac{1}{2}$ x 200 x h = 2000

$$h = \frac{2000 \times 2}{200}$$

= 20 mm

6 Find the area of an equilateral triangle whose side is 5 cm.

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 \text{ unit}^2$$

$$= \frac{1.732}{4} \times 5 \times 5$$

= 10.825 cm²

7 Calculate its perimeter if one side of an equilateral triangle is 55 mm long.

Side = 55 mm

Perimeter = ?

P = 3a unit

= 3 x 55

= 165 mm

8 Find the area of the triangle having its sides 9cm, 10cm and 12 cm.

$$\text{Semi Perimeter} = \frac{a+b+c}{2} \text{ unit}$$

$$= \frac{9+10+12}{2} = \frac{31}{2}$$

= 15.5 cm

$$\text{Area A} = \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2$$

$$= \sqrt{15.5(15.5-9)(15.5-10)(15.5-12)}$$

$$= \sqrt{15.5 \times 6.5 \times 5.5 \times 3.5}$$

$$= \sqrt{1939.4375}$$

= 44.03 cm²

9 Find the cost of polishing on both sides of a triangular metal plate has sides 60 cm, 50 cm and 20 cm at the rate of Rs.1.35 per 100 cm²

$$\text{Semi Perimeter} = \frac{a+b+c}{2} \text{ unit}$$

$$= \frac{60+50+20}{2} = \frac{130}{2}$$

= 65 cm

$$\text{Area A} = \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2$$

$$= \sqrt{65(65-60)(65-50)(65-20)}$$

$$= \sqrt{65 \times 5 \times 15 \times 45}$$

= 468.4 cm²

Area of polish on both sides = 2 x 468.4

= 936.8 cm²

Cost of polish per 100 cm² = Rs. 1.35

∴ Cost of polish is 936.8 cm² $= \frac{936.8}{100} \times 1.35$

= Rs. 12.65

10 Find the area of the right angled triangle with base 20 cm and height 8 cm.

Base b = 20 cm

Equal sides or slant height = 8 cm

Area A = ?

Area A = $\frac{1}{2}$ x base x height unit²

= $\frac{1}{2}$ x 20 x 8

= 80 cm²

11 Find the area of the right angled triangle if the sides containing the right angle being 10.5 cm and 8.2 cm.

Area A = $\frac{1}{2}$ x base x height unit²

= $\frac{1}{2}$ x 10.5 x 8.2

= 43.05 cm²

12 Calculate the perpendicular height of the triangle if the area of the right angled triangle is 19.44 m^2 and its one of the adjacent side containing the right angle being 5.4 m.

$\frac{1}{2}$ x base x height $unit^2$ = Area A

$\frac{1}{2}$ x 5.4 x h = 19.44

$$h = \frac{19.44 \times 2}{5.4}$$

= 7.2 m

13. Calculate the base of a right angled triangle having an area of 15 cm^2. If its height is 3.5 cm.

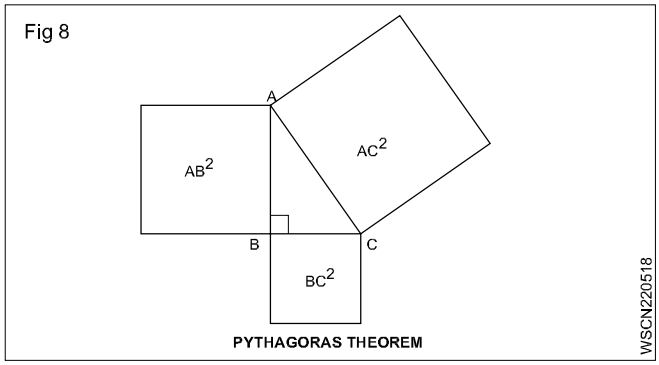
$\frac{1}{2}$ x base x height $unit^2$ = Area A

$\frac{1}{2}$ x b x 3.5 = 15

$$b = \frac{15 \times 2}{3.5}$$

= 8.57 cm

Pythagoras theorem



In a right angled triangle the area of the square drawn with the hypotenuse as the side is equal to the sum of the areas of the squares drawn with the other two sides.

$\angle B$ = 90°

AC = Hypotenuse

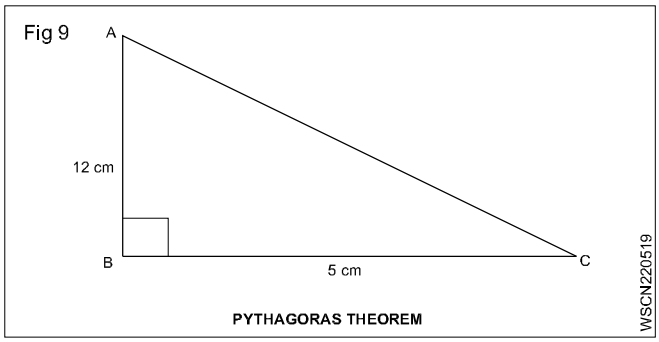
AB & BC = Adjacent sides

As per pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore$ $AC = \sqrt{AB^2 + BC^2}$

1 Calculate the hypotenuse of a right angled triangle whose base is 5 cm and height 12 cm.



As per pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 12^2 + 5^2$

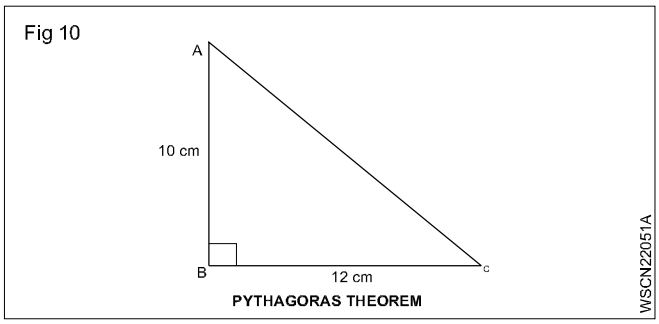
$= 144 + 25$

$= 169$

$AC = \sqrt{169}$

= 13 cm

2 What is the length of the hypotenuse of a right angled triangle, when the sides containing the right angles are 10 cm and 12 cm.



As per pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 10^2 + 12^2$

$= 100 + 144$

$= 244$

$AC = \sqrt{244}$

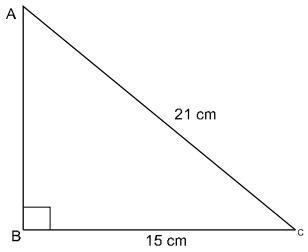
= 15.62 cm

3 Find the height of a right angled triangle whose base is 15 cm and hypotenuse is 21 cm.

As per pythagoras theorem,

$AB^2 + BC^2 = AC^2$

Fig 11



PYTHAGORAS THEOREM

WSCN22051B

$AB^2 + 15^2 = 21^2$

$AB^2 = 441 - 225$

$= 216$

$AB = \sqrt{216}$

$= 14.7$ cm

Assignment

I

1 Find the area of a triangle whose base is 85.4 mm and its height 29 mm respectively.

2 The area of a triangle is 30 sq. cm. Its base is 10 cm. Find its height.

3 Calculate the base of a triangle having an area of 80 cm^2 and height 8 cm.

4 Calculate the height of triangle whose area is 160 cm^2 and base is 20 cm.

II

1 Find the area of an isosceles triangle whose base is 16 cm long and each of the other two sides are 10 cm long.

2 Find the area of an isosceles triangle whose side is 7 cm and base is 5 cm.

3 Find the area of an isosceles triangle whose side is 10 cm and base is 8 cm.

III

1 Find out the area of an equilateral triangle whose base is 2.8 cm.

2 Find the area of an equilateral triangle whose sides are 8 cm each.

3 Find the area of an equilateral triangle whose one side is 64 mm.

4 Find the area of a triangle whose all sides are equal and sum of the three sides is equal to 12 cm.

IV

1 Find the area of a triangle whose sides are 6 cm, 7 cm and 9 cm.

2 Calculate the area of the triangle if sides of a triangle are 3 cm, 4 cm and 6 cm.

3 Find the area of a triangle whose sides are 20 cm, 16 cm and 10 cm

4 Find the area and perimeter of the triangle if the three sides of a triangle are 5 mm, 12 mm and 13 mm respectively.

5 Find the area and perimeter of the triangle if the sides of a triangle are 15 mm, 17 mm and 8 mm respectively.

V

1 Find the area of a right angled triangle whose base is 15 cm and perpendicular height is 21 cm.

2 Find the area of a right angled triangle has its base side 60 mm and height 75 mm.

3 Find the area of a right angled triangle the adjacent sides to the right angle being 13.7 cm and 9.2 cm.

4 Calculate the height of triangle whose area is 60 cm^2 and base is 10 cm.

5 Calculate the height of triangle whose area is 160 cm^2 and base is 20 cm.

6 Calculate the base of a triangle having an area of 80 cm^2 and height is 8 cm.

VI

1 What is the length of the third side in a right angled triangle the two small sides are 30 cm and 40 cm.

2 Find the length of the hypotenuse of a right angled triangular frame having 60 cm base and 18 cm height.

3 Find the height of an equilateral triangle whose side is 60 cm.

4 ABC is a right angled triangle. If AB = 15 cm and BC = 17 cm. Find the length of AC.

5 Find out the length of AC in a right angled triangle ABC, AB=30 cm, BC = 40 cm.

6 Find the height of a right angled triangle whose base is 20 cm and hypotenuse is 30 cm.

Circle

It is the path of a point which is always equal from its centre is called a circle.

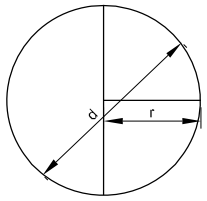
r = radius of the circle

d = diameter of the circle

$$\pi = \frac{22}{7} = 3.14$$

Area of the circle = πr^2

Fig 1



PROPERTIES OF CIRCLE

WSCN220611

(or) $= \frac{\pi}{4} d^2$ unit^2

Circumference of the circle $2\pi r$ (or) πd unit

Examples

1. Find the area of a circle whose radius is 1.54 m. Also find its circumference.

radius r = 1.54 cm

Area A = ?

Circumference C = ?

$A = \pi r^2$ unit^2

$= \frac{22}{7} \times 1.54 \times 1.54$

$= 7.4536$ m^2

$C = 2\pi r$ unit

$= 2 \times \frac{22}{7} \times 1.54$

= 9.68 m

2. Find out the circumference if the area of a circular shape of land is 616 m^2.

$A = \pi r^2$ unit^2

$r^2 = \frac{616}{\pi}$

$= \frac{616 \times 7}{22}$

$= 196$

$r = \sqrt{196}$

$= 14$ m

$C = 2\pi r$ unit

$= 2 \times \frac{22}{7} \times 14$

= 88 m

3. Find the side of square into which it can be bent if a wire is in the form of a circle of radius 49 cm.

radius of circle r = 49 cm

side of square = ?

Perimeter of the square = Perimeter of the circle

$4a = 2\pi r$

$4a = 2 \times \frac{22}{7} \times 49$

$4a = 308$

$a = \frac{308}{4}$

= 77 cm

4. Find its radius if the difference between the circumference and diameter of a circle is 28 cm.

Circumference - Diameter = 28 cm

$2\pi r - d = 28$

$2\pi r - 2r = 28$

$2r(\pi - 1) = 28$

$2r\left(\frac{22}{7} - 1\right) = 28$

$2r\left(\frac{22-7}{7}\right) = 28$

$2r \times \frac{15}{7} = 28$

$r = \frac{28 \times 7}{15 \times 2}$

= 6.53 cm

5. What is the side of the largest square cut out from a circle of 50 cm dia.?

Diagonal of a square = Diameter of the circle

$\sqrt{2}a = 50$

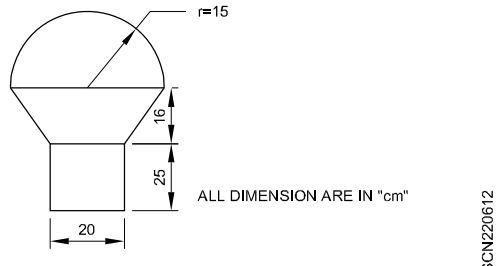
$a = \frac{50}{\sqrt{2}}$

$= \frac{50}{1.414}$

= 35.36 cm

6. Calculate the area of the figure given below.

Fig 2



PROPERTIES OF CIRCLE

Area of rectangle = lb unit^2

= 25 x 20 cm^2

= 500 cm^2

Area of Trapezium = $\frac{1}{2}$ x (a + b) h

= $\frac{1}{2}$ x (30 + 20) 16 cm^2

= $\frac{1}{2}$ x 50 x 16 cm^2

= 400 cm^2

Area of Semi circle = $\frac{\pi r^2}{2}$ unit^2

= π x 15^2 x $\frac{1}{2}$ cm^2

= 353.57 cm^2

Area of the figure = 500 + 400 + 353.57

= 1253.57 cm^2

7. Find the area of remaining steel plate if in a rectangular steel plate 16 cm x 12 cm, there are 6 holes each 4 cm in diameter.

Area of a rectangular plate = length x breadth unit^2

= 16 x 12

= 192 cm^2

No. of holes = 6

Radius of hole = 2 cm

Area of 6 holes = 6 x πr^2 unit^2

= 6 x $\frac{22}{7}$ x 2 x 2 unit^2

= 75.43 cm^2

Area of remaining plate = 192 - 75.43

= 116.57 cm^2

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Semi circle

A semi circle is a sector whose central angle is 180º.
Length of arc of semi circle.

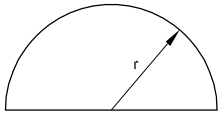
Length of arc ℓ = $2\pi r$ x $\frac{180}{360}$

= $2\pi r$ x $\frac{1}{2}$

= πr unit

Area of semi circle = $\frac{\pi r^2}{2}$ Sq. units

Fig 3



SEMI CIRCLE

Perimeter of a semi circle = $\frac{2\pi r}{2}$ + 2r

= πr + 2r

= r (π + 2) unit

Examples

1. Calculate the circumference and area of a semi circle whose radius is 6 cm.

radius r = 6 cm

Area A = ?

Circumference c = ?

A = $\frac{\pi r^2}{2}$ unit^2

= $\frac{22}{7}$ x $\frac{1}{2}$ x 6^2

Area (A) $= \frac{22}{7} \times \frac{1}{2} \times 36$

$= \frac{396}{7} = 56.57 \text{ cm}^2$

Perimeter of a semicircle $= 6(\frac{22}{7} \times 2)$

$= 6\left(\frac{22+14}{7}\right)$

$= 6 \times \frac{36}{7}$

$= \frac{216}{7}$

= 30.86 cm

2. From the figure given below ABCD is a steel plate, a semi circular plate of radius 50 mm has been prepared by gas cutting. Find the waste area.

Fig 4

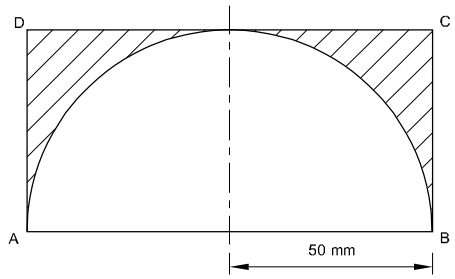


Plate length AB = 100 mm

Breadth BC = 50 mm

Radius = 50 mm

Waste area = Plate area - Area of semi circle

$= lb - \frac{\pi r^2}{2}$

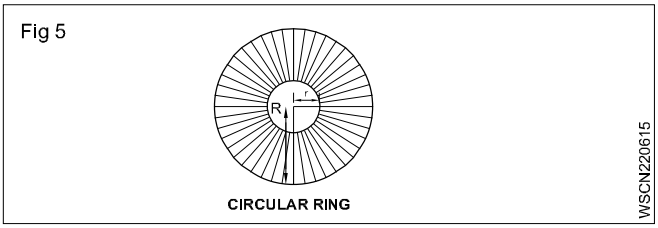
$= 100 \times 50 - \frac{22 \times 50 \times 50}{7 \times 2}$

$= 5000 - 3928.57$

$= 1071.43 \text{ mm}^2$

Circular ring

Fig 5



R = Outer radius of circular ring

r = Innner radius of circular ring

Area of circular ring = $\pi(R^2 - r^2)$ unit^2

or

A = π (R + r) (R - r) unit^2

1 Calculate the area of cross section of pipe having outside dia of 17 cm and inside dia of 14 cm.

Given:

Outer dia of pipe = 17 cm

Outer radius of pipe (R) = $\frac{17}{2}$ = 8.5 cm

Inner dia of pipe = 14 cm

Inner radius of pipe (r) = = 7 cm

To find:

Area of cross section of pipe = ?

Solution:

Area of cross section of pipe $= \pi (R + r) (R - r)$ unit^2

$= \pi (8.5 + 7) (8.5 - 7)$

$= \frac{22}{7} \times 15.5 \times 1.5 \text{ cm}^2$

$= 73 \text{ cm}^2$

2 Find the distance between the boundaries and the area of the circular ring, if the circumference of two concentric circle are 134 cm and 90 cm.

Given:

Circumference of outer circle = 134 cm

Circumference of inner circle = 90 cm

To find:

Distance between the circles = ?

Area of circular ring = ?

Solution:

Circumference of outer circle = 134 cm

$2\pi R = 134$ cm

$R = \frac{134}{2\pi} = 21.32\text{cm}$

Circumference of inner circle = 90 cm

$2\pi r = 90$ cm

$$r = \frac{90}{2\pi} = 14.32 \text{ cm}$$

Distance between the circle = R - r

= 21.32 - 14.32 cm

= 7 cm

Area of circular ring = π (R + r) (R - r) unit^2

= π (21.32 + 14.32) (21.32 - 14.32) cm^2

$$= \frac{22}{7} \text{ x } 35.64 \text{ x } 7 \text{ cm}^2$$

= 784.08 cm^2

3 Calculate the inner and outer diameter of the circular ring. If the area of a circular ring is 176 cm^2 and width of the circular ring is 4 cm.

Given:

Area of circular ring = 176 cm^2

width = 4 cm

To find:

Outer diameter = ?, Inner diameter = ?

Solution:

Inner radius (r) = x cm

Outer radius (R) = inner radius + width

= x + 4 cm

π (R + r) (R - r) = area

π (x + 4 + x) (x + 4 - x) = 176 cm^2

π (2x + 4) (4) = 176 cm^2

$$\frac{22}{7} \text{ x } 4 \text{ x } (2x + 4) = 176 \text{ cm}^2$$

$$\frac{88}{7} \text{ x } (2x + 4) = 176 \text{ cm}^2$$

$$2x + 4 = \frac{176 \times 7}{88} = 14$$

$2x = 14 - 4 = 10$

$$x = \frac{10}{2} = 5 \text{ cm}$$

inner radius (r) = x = 5 cm

outer radius (R) = x + 4 = 5 + 4 = 9 cm

inner diameter = 2 x 5 = 10 cm

outer diameter = 2 x 9 = 18 cm

4 Calculate the length of material required , if a piece of 12mm dia bar is to be bent round to form a ring 150 mm inside dia.

Given:

dia of bar = 12 mm

inner diameter = 150 mm

To find:

length of bar = ?

Solution:

Inner diameter = 150 mm

Outer diameter = dia of bar + inner diameter + dia of bar

= 12 + 150 + 12 = 174 mm

$$\text{Average diameter} = \frac{\text{inner diameter} + \text{outer diameter}}{2}$$

$$= \frac{150 + 174}{2} = \frac{324}{2} = 162 \text{ mm}$$

$$\text{Average radius} = \frac{162}{2} = 81 \text{ mm}$$

Length of the material required = average circumference

= 2πr unit

= 2 x π x 81 mm

= 509 mm

5 A wire can be bend in the form of a circle of radius 56 cm. If it is bend in a form of a square, find the side.

Given:

Radius of circle = 56 cm

To find:

Side of square = ?

Solution:

Radius of circle = 56 cm

Circumference of circle = 2πr unit

= 2π x 56 cm

Side of square = x cm

Wire can be bend from the form of round to square

Perimeter of square = circumference of circle

4 x a = 352 cm

$$a = \frac{352}{4} = 88 \text{ cm}$$

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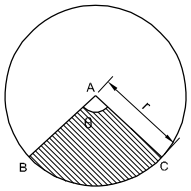
Sector of Circle

Fig 6



SECTOR OF CIRCLE

WSCN220616

θ = Angle of sector of circle

l = Arc length

r = radius

Length of Arc $\ell = \frac{\theta}{360^0} \times 2\pi r$ unit

Perimeter P = 2r + ℓ unit

Area = $\frac{\theta}{360^0} \times \pi r^2$ unit^2

or

$A = \frac{\ell r}{2}$ unit^2

1 Find the perimeter and area of a sector of circle of radius 7 cm and its angle is 120^0.

Given:

Angle of sector of circle = 120^0

Radius = 7 cm

To find:

Perimeter = ? , Area = ?

Solution:

Length of arc (ℓ) = $\frac{\theta}{360^0} \times 2\pi r$ unit

$= \frac{120}{360} \times 2 \times \frac{22}{7} \times 7$ cm

= 14.67 cm

Perimeter = 2r + ℓ unit

= 2 x 7 + 14.67 cm

= 28.67 cm

Area = $\frac{\theta}{360^0} \times \pi r^2$ unit^2

$= \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 7$ cm^2

= 51.33 cm^2

2 Find the radius of the circle if the angle is 60^0 and the area of a sector of a circle is 144 cm^2,

Given:

Area of sector of circle (A) = 144 cm^2

Angle of sector of circle $\theta = 60^0$

To find:

Radius of circle = ?

Solution:

Area (A) = $\frac{\theta}{360^0} \times \pi r^2$ unit^2

144 = $\frac{60^0}{360^0} \times \frac{22}{7} \times r^2$ cm^2

r^2 = 274.91 cm^2

r = $\sqrt{274.91}$ = 16.58 cm

3 Find the area of the sector whose angle is 105^0, and the perimeter of sector of circle is 18.6 cm.

Given:

Perimeter of a sector of a circle = 18.6 cm

Angle of sector of circle = 105^0

To find:

Area = ?

Solution:

Length of Arc (ℓ) = $\frac{\theta}{360^0} \times 2\pi r$ unit

$\ell = \frac{105^0}{360^0} \times 2 \times \frac{22}{7} \times r$

= 1.83r

Perimeter (P) = ℓ + 2r unit

18.6 = 1.83r + 2r

3.83r = 18.6 cm

$r = \frac{18.6}{3.83}$ = 4.86 cm

Area A = $\frac{\theta}{360^0} \times \pi r^2$ unit^2

$= \frac{105^0}{360^0} \times \frac{22}{7} \times (4.86)$ cm^2

= 21.65 cm^2

4 Find the area, if the radius is 12.4 cm and the perimeter of a sector of a circle is 64.8 cm.

Given:

Perimeter P = 64.8 cm

Radius r = 12.4 cm

To find:

Area A = ?

Solution:

Perimeter P = ℓ + 2r unit

ℓ = P - 2r unit

= 64.8 - 2 (12.4) cm

= 64.8 - 24.8 = 40 cm

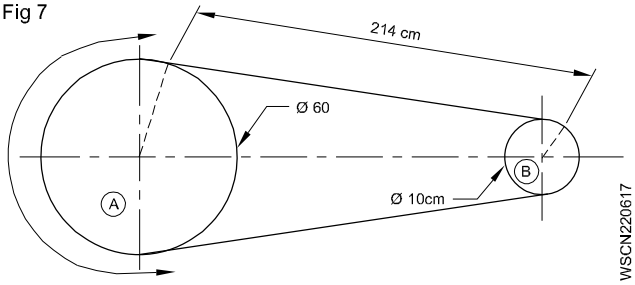
Area A = $\frac{\ell r}{2}$ unit^2

= $\frac{40 \times 12.4}{2}$ = 248 cm^2

5 Find out the length of the belt , if the arrangement of a belt is shown in the figure below.

Solution:

Fig 7



Length $\ell_A = \frac{\theta}{360^0} \times 2\pi r$ unit

$= \frac{210^0}{360^0} \times 2 \times \frac{22}{7} \times 30 = 110$ cm

Length $\ell_B = \frac{\theta}{360^0} \times 2\pi r$ unit

$= \frac{105^0}{360^0} \times 2 \times \frac{22}{7} \times 5 = 91.7$ cm

= $\ell_A + \ell_B + 2 \times 214$ cm

= 110 + 9.17 + 428 cm

= 547.17 cm

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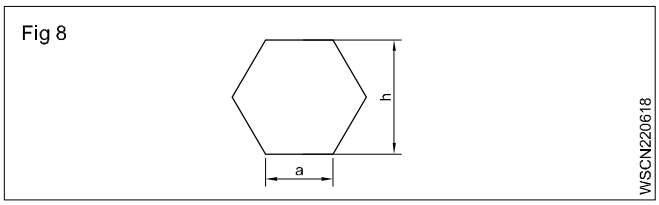
Hexagon

Fig 8



Side = a unit

Perimeter P = 6a unit

Area A = $6 \times \frac{\sqrt{3}}{4} \times a^2$ units^2 (Area of 6 equilateral triangle)

DAF (Distance Across Flats) = $\sqrt{3} \times a$ unit

DAC (Distance Across Cormers) = 2 x a unit

Example 1

Find out the perimeter, area, DAF and DAC of a regular hexagon whose side is 2cm.

(DAF - Distance Across Flats)

(DAC - Distance Across Corners)

Given: Side of hexagon (a) = 2cm

To Find: P = ?, A = ?, DAF = ?, DAC = ?

Solution:

Perimeter of hexagon (P) = 6a unit

= 6a unit = 6 x 2 cm = 12 cm

Area of hexagon A = $6 \times \frac{\sqrt{3}}{4} \times a^2$ unit^2

= $6 \times \frac{1.732}{4} \times 2^2$

= 10.392 cm^2

DAF (Distance Across Flats) = $\sqrt{3} \times a$ unit

= $\sqrt{3} \times 2$ = 1.732 x 2

= 3.464 cm

DAC (Distance Across Corners) = 2 x a unit

= 2 x 2 = 4 cm

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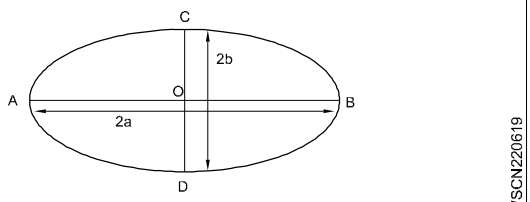
Ellipse

Fig 9



Major axis AB = 2a

Half of Major axis OB = a,

Minor axis CD = 2b

Half of Minor axis OC = b

Area of ellipse A = x a x b unit^2

Perimeter of ellipse P = $2\pi\sqrt{\frac{(a^2+b^2)}{2}}$ unit

Example 1

Find its area and perimeter, if the major and minor axis of an ellipse are 12 cm and 8 cm respectively.

Solution:

Major axis 2a = 12 cm

$a = \frac{12}{2} = 6$ cm

Minor axis 2b = 8 cm

$b = \frac{8}{2} = 4$ cm

Area A = π x a x b unit^2

$= \frac{22}{7}$ x 6 x 4 cm^2

= 75.43 cm^2

Perimeter (P) $= 2\pi\sqrt{\frac{(a^2+b^2)}{2}}$ unit

$= 2\times\frac{22}{7}\sqrt{\frac{(6^2+4^2)}{2}}$ unit

$= 2\times\frac{22}{7}\sqrt{\frac{36+16}{2}}$ unit

$= 2\times\frac{22}{7}\times\sqrt{26}$

$= 2 \times \frac{22}{7}$ x 5.1 = 32.06 cm

Assignment

Circle

1 Find the circumference and area of a circle whose radius is 10.00 metre.

2 Find its diameter if the area of a circle is 330 cm^2.

3 Find its radius if area of a circle is 498 m^2.

4 Find its area if the circumference of a circle is 50 cm.

5 Find its area if the circumference of a circle is 44 cm.

6 Find out the area and circumference of a circle of diameter is 50 cm.

7 A wire is in the form of a circle whose radius is 42 cm. Find the side of that square which can be made by bending the same wire.

8 A square of side 22 cm is made from a wire. Calculate the diameter of circle which will be made from the same length of wire as that of square.

9 Find its radius if the difference between the circumference and diameter of a circle is 30 cm.

10 Calculate the side of the largest square that can be obtained if a 150 mm dia. round bar is milled to a square bar.

11 What is the maximum size of square which can be cut from a circular sheet of diameter 100 mm?

12 From a brass sheet 270 cm x 100 cm. Calculate how many pieces of size 15 cm x 10 cm may be cut.

13 Find the area of remaining plate if in a 48 cm x 18 cm rectangular plate there are 5 holes of 4 cm diameter.

14 Find the area of remaining steel plate if in a rectangular steel plate 36 cm x 24 cm. There are 54 holes of 4 cm in diameter.

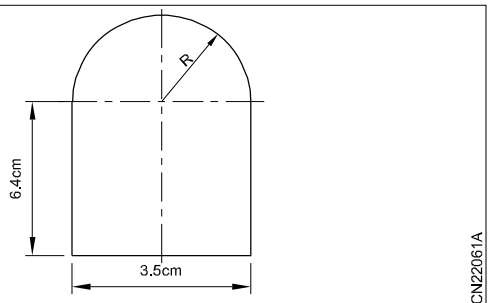
15 Find the radius of circle if a rectangle with sides 14 metre length and 11 metre breadth has area equal to that of circle.

Semi circle

1. Calculate the circumference and area of semi circle whose radius is 14 cm.

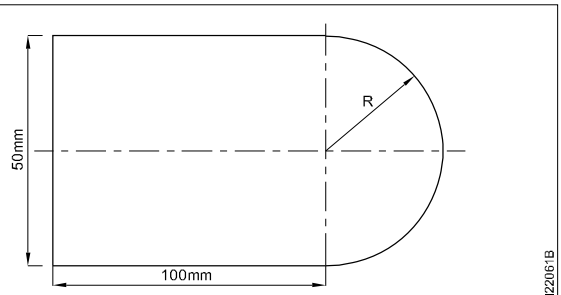
2. Find the area of the figure given below.

Fig 10



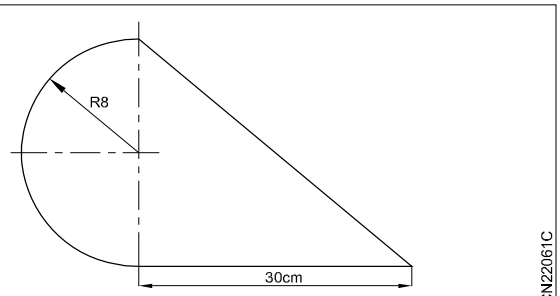
3. Calculate the area of the given figure.

Fig 11



4. Calculate the area of the figure given below.

Fig 12



Circular ring

1 Find out area of a ring washer, whose inner radius and outer radius are 13 cm and 15 cm respectively.

2 Find the area of a ring portion of a washer whose outer dis is 30 m and inner dis is 20 m. Also calculate the difference between the circumference of circles.

3 Find the thickness of the pipe and area of cross section of the pipe, if the outside and inside circumference of the steel pipe is 70 cm and 45 cm.

4 Calculate the inner and outer diameter of the washer.if the cross sectional area of a washer is 264 cm^2 and the width of the washer is 2cm.

5 What length of rod did you require to make a ring of 300cm inside dia out of 12 mm round bar?

Sector of circle

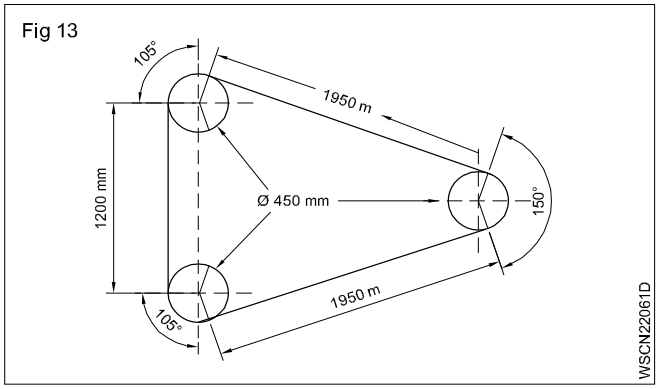
1 Find the perimeter and area of a sector of a circle of radius 5cm and its angle is 96^0.

2 Find the radius of the circle if the angle is 90^0 and the area of sector of a circle is 196 cm^2.

3 Find the area of a sector of a circle whose perimeter is 82.4 mm and radius is 16.2 mm.

4 Find out the length of the saw blade, if the arrangement of a band saw is shown in the figure below.

Fig 13



Hexagon

1 Find out the Area, perimeter, DAF,and DAC of hexagon of side 4cm.

2 Find the area of cross section of a regular hexagon rod whose side is 7.5 cm.

3 Find out the size across the flats of a hexagonal piece having 15 mm each sides.

4 Find out the distance across the flats, if the distance across the c orners of a hexagonal bar 40 mm.

5 What will be the area of a largest hexagon which is inscribed in a circle of radius 10 cm?

Ellipse

1 Find the area of the biggest ellipse that can be inscribed in a rectangle of length 18 cm and breadth 12 cm. Also calculate its perimeter.

2 How much fenching will be required to enclose an elliptical plot of ground the axes of the ellipse being 200 and 170 meter respectively.

Cube

All sides of cube are same i.e length,breadth and height have same value. It is bounded by six equal square faces.

Volume of cube = side x side x side

$= a^3$ unit^3

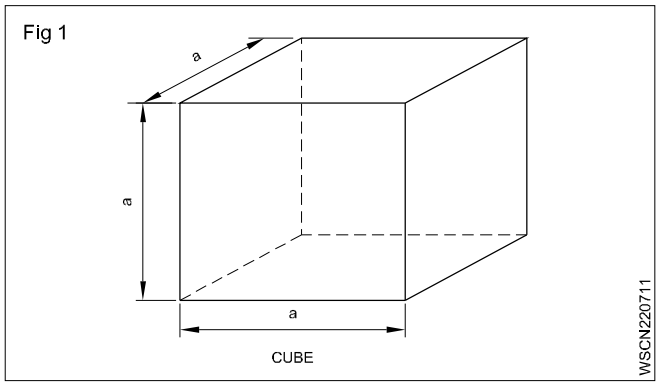
Lateral surface area = $4a^2$ unit^2

Total surface area = 6 x side x side

$= 6a^2$ unit^2

Rectangular solid (or) cuboid

Rectangular soild is bounded by six rectangular surfaces and opposite surfaces are equal and parallel to each other.



Volume of rectangular solid

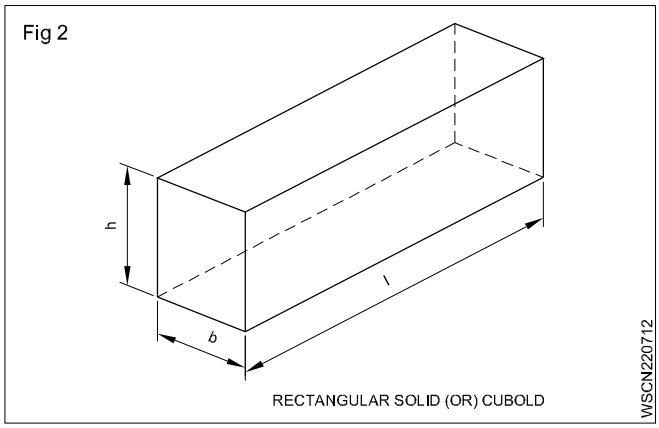
= Length x breadth x height

= l . b . h unit^3

Lateral surface area = 2h(l+b) unit^2

Total surface area = 2lb + 2bh + 2hl

$= 2(lb+bh+hl)$ unit^2



l = length

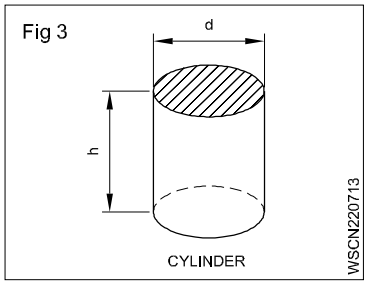
b = breadth

h = height

Cylinder

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$



Curved area of cylinder = $2\pi rh$ unit^2

Total surface area of cylinder = $2\pi r(h+r)$ unit^2

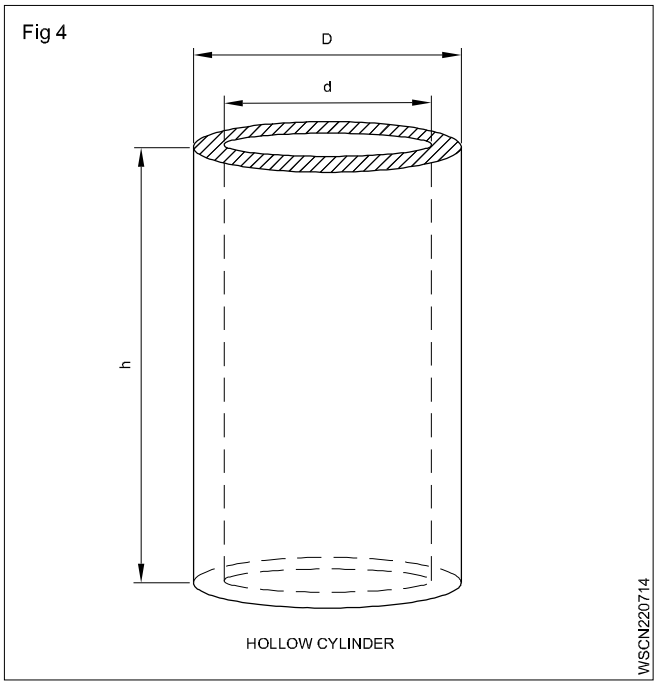
r = Radius of base

d = Diameter of base

h = Height of cylinder

Hollow cylinder

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.



Volume of hollow cylinder = $\pi (R^2 - r^2) h$ (or)

$= \pi (R + r) (R - r) h$ (or)

$= \frac{\pi}{4} (D^2 - d^2) h$ unit^3

$= \frac{\pi}{4}(D + d)(D - d)h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

$\therefore$ TSA : $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

Mean dia = $\frac{D-d}{2}$

If thickness given then:

Volume of hollow cylinder = π x mean dia x thickness x height

Finding out volumes of solids

The space occupied by a body is known its volume. The volume of a body indicates the capacity to hold substance in it.

The general form of Lateral surface area Total surface area and Volume is :

Lateral surface area = perimeter of the base x height

Total surface area = LSA + 2 (base area)

Volume = Area of base x height

Important and commonly used solids are described below one after another:

Cube

All sides of cube are same i.e length,breadth and height have same value. It is bounded by six equal square faces.

Volume of cube = side x side x side

$= a^3$ unit^3

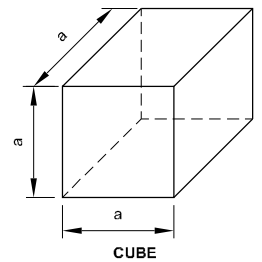
Lateral surface area = $4a^2$

Total surface area = 6 x side x side

$= 6a^2$ unit^2

Diagonal d $= \sqrt{3}a$ unit where $\sqrt{3} = 1.732$

Fig 5



WSCN220715

1 Find the diagonal, lateral surface area,, total surface area and volume of a cube of side 4.5 cm.

side a = 4.5 cm

diagonal d $= \sqrt{3}a$ unit

= 1.732 x 4.5

= 7.794 cm

L.S.A = $4a^2$ unit^2

= 4 x 4.5 x 4.5

= 81 cm^2

T.S.A = $6a^2$ unit

= 6 x 4.5 x 4.5

= 121.5 cm^2

V = a^3 unit^3

= 4.5 x 4.5 x 4.5

= 91.125 cc.

2 Calculate volume of a cube where side is 9 cm

a = 9 cm

V = ?

V = a^3

= 9 x 9 x 9

= 729 cm^3

3 Find out side of the cube if a cube has volume of 3375cm^3.

V = 3375 cm^3

a = ?

a^3 = 3375

$a = \sqrt[3]{3375}$

$= \sqrt{3x3x3x5x5x5}$

= 3 x 5

= 15 cm

4 Find the side of a cube, if its surface area is 216 cm^2

Surface area = T.S.A = 216 cm^2

$6a^2 = 216$

$a^2 = \frac{216}{6}$

= 36

$a = \sqrt{36}$

= 6 cm

5 Find the side of the square tank, if its height is 2 metre and has the capacity to hold 50,000 litre of water.

Height of square shape tank (h) = 2 m

Capacity = 50,000 litre

1000 litre = 1 m^3

50,000 Litre = $\frac{50000}{1000}$

$= 50\ m^3$

Capacity of tank $= 50\ m^3$

$a^2 \times h = 50$

$a^2 \times 2 = 50$

$a^2 = \frac{50}{2} = 25\ m^2$

$a = \sqrt{25} = 5\ m$

Side of the square tank = 5 m

Rectangular solid (or) cuboid

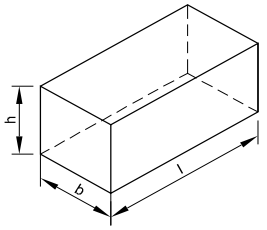
Rectangular soild is bounded by six rectangular surfaces and opposite surfaces are equal and parallel to each other.

Volume of rectangular solid

= Length x breadth x height

= l . b . h $unit^3$

Fig 6



RECTANGULAR (OR) CUBOID

WSCN220716

Lateral surface area = 2h(l+b)

Total surface area = 2lb + 2bh + 2hl

= 2(lb+bh+hl) $unit^3$

Examples

1 Find its volume and T.S.A if a tank is 20 m long, 15 m broad and 12 m high.

l = 20m

b = 15 m

h = 12 m

v = ?

T.S.A = ?

Volume v = lbh $unit^3$

= 20 x 15 x 12

= 3600 m^3

T.S.A = 2(lb + bh + hl) $unit^2$

= 2((20 x 15) + (15 x 12) + (20 x 12))

= 2 (300 + 180 + 240)

= 1440 m^2

2 Find out its height if the cross section is 260 mm length and 180 mm wide rectangular and the capacity of a fuel tank is 10500 cm^3.

l = 260mm = 26 cm

b = 180 mm =18 cm

v = 10500 cm^3

h = ?

l.b.h = volume

26 x 18 x h = 10500

$h = \frac{10500}{26 \times 18}$

= 22.44 cm

3 How many litres of water it can store if a water tank has the following dimensions length = 1 metre, width = 0.8 metre and height = 1.2 metre?

Volume = l x b x h $unit^3$

= 1 x 0.8 x 1.2

= 0.96 m^3 [1 m^3 = 1000 litres]

= 0.96 x 1000

= 960 litres of water can store in the tank.

4 Find its volume if the base of a prism is a rectangle having 5m length, 4m breadth and the height of the prism is 15m.

The base of prism is rectangle

Area of base = length x breadth

= 5 x 4

= 20 square m

Volume of prism = Area of base x Height

= 20 x 15

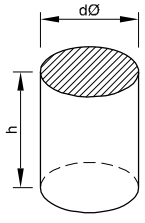
= 300 cubic metres

Cylinder

This is a prism whose top and bottom surfaces are equal circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$

Fig 7



CYLINDER

WSCN220717

Curved area of cylinder = $2\pi rh$

Total surface area of cylinder = $2\pi r(h+r)$

r = Radius of base

d = Diameter of base

h = Height of cylinder

Examples

1 Find the volume and total surface are of a cylinder having 9cm diameter and 15 cm height.

diameter = 9 cm

radius r = 4.5 cm

height h = 15 cm

Volume V = ?

T.S.A = ?

$V = \pi r^2 h$ unit^3

$= \frac{22}{7} \times 4.5 \times 4.5 \times 15$

$= 954.4$ cm^3

T.S.A $= 2\pi r(h+r)$ unit^2

$= 2 \times \frac{22}{7} \times 4.5\ (15 + 4.5)$

$= 2 \times \frac{22}{7} \times 4.5 \times 19.5$

= 551.4 cm^2

2 Calculate the radius if the curved surface area of a cylindrical roller is 48π cm^2 and the roller is 10 cm long

C.S.A $= 48\pi$ cm^2

length = 10 cm

radius = ?

$2\pi rh = 48\pi$

$2 \times \pi \times r \times 10 = 48\pi$

$r = \frac{48 \times \pi}{2 \times \pi \times 10}$

= 2.4 cm

3 Find its radius if the volume of a cylinder is 5544 cm^3 and its height is 16 cm.

$\pi r^2 h = v$

$3.14 \times r^2 \times 16 = 5544$

$r^2 = \frac{5544}{3.14 \times 16}$

$r^2 = \frac{5544}{50.24}$

$= 110.35$

$r = \sqrt{110.35}$

= 10.5 cm

4 Find the diameter of the tank if the volume of a circular tank is 68.46 m3, its height is 2 m.

$\pi r^2 h = 68.46$

$r^2 = \frac{68.46}{3.14 \times 2}$

$r^2 = 10.9$

$r = \sqrt{10.9}$

$= 3.3$ m

diameter $= 2r$

$= 2 \times 3.3$

= 6.6 m

5 A cylindrical vessel is to be made of 3 metre long and 1.9994 metre diameter. Calculate its surface area, if it is in a closed form on one end.

h = 3m

d = 1.9994 m

r = 0.9997 m

T.S.A = C.S.A + Base area

$= 2\pi rh + \pi r^2$

$= (2 \times \frac{22}{7} \times 0.9997 \times 3) + (\frac{22}{7} \times 0.99997^2)$

$= 18.85 + 3.14$

= 21.99 m^2

6 How many litres of water a cylinder of radius 75 cm and height 100 cm can hold.

$V = \pi r^2 h$ unit^3

$= 3.142 \times 75 \times 75 \times 100$

$= 1767375$ cm^3

$= \frac{1767375}{1000}$ [1000 cc = 1 litre]

= 1767.375 litres.

7 Calculate the height of cylindrical tin if a closed rectangular box 40 cm long, 30 cm wide and 25 cm deep has the same volume as that of cylinder tin of radius 17.5 cm.

Volume of cylinder = Volume of rectangular box

$\pi r^2 h = l \times b \times h$

$\frac{22}{7} \times 17.5 \times 17.5 \times h = 40 \times 30 \times 25$

$h = \frac{40 \times 30 \times 25 \times 7}{22 \times 17.5 \times 17.5}$

$= \frac{210000}{6737.5}$

= 31.17 cm

8 An oxygen cylinder is 15 cm in diameter and 100 cm in length. It is filled with gas under pressure so that every cm^3 of the cylinder contains 120 cm^3 of gas. How much cc of oxygen does this hold?

Volume of cylinder $= \pi r^2 h$ unit^3

$$= \frac{22}{7} \times 7.5 \times 7.5 \times 100$$

$$= 17678.57 \text{ cm}^3$$

Gas contain in 1 cm^3 = 120 cm^3 of gas

Gas contain in 17678.57 cm^3 = 17678.57 x 120

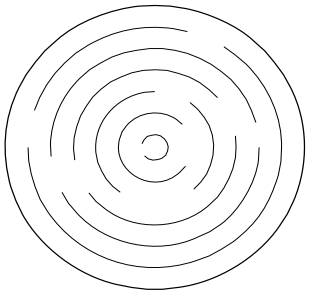
$$= 2121428 \text{ cm}^3$$

Volume of oxygen = 2121428 cc.

Sphere

Sphere is a solid circular body.

Fig 8



SPHERE

WSCN220718

Volume of sphere = $\frac{4}{3}\pi r^3$ or

$$= \frac{\pi}{6} d^3 \text{ unit}^3$$

Total surface area of sphere = $4\pi r^2$ $unit^2$

Where r = Radius of sphere

d = Diametre of sphere

Radius = $\frac{1}{2}$ of diameter

Examples

1 Find the volume and surface area of a sphere of 3 cm radius.

$$V = \frac{4}{3}\pi r^3 \text{ unit}^3$$

$$= \frac{4 \times 22 \times 3 \times 3 \times 3}{3 \times 7}$$

$$= 113.1 \text{ cm}^3$$

Total Surface Area $= 4\pi r^2$ $unit^2$

$$= 4 \times \frac{22}{7} \times 3 \times 3$$

= 113.1 cm^2

2 Find the diameter of sphere having volume of 15625 cc.

$$\frac{4}{3}\pi r^3 = \text{Volume}$$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = 15625$$

$$r^3 = \frac{15625 \times 3 \times 7}{4 \times 22}$$

$$= \frac{328125}{88}$$

$$= 3728.69$$

$$r = \sqrt[3]{3728.69}$$

= 15.51 cm

diameter = 2 x radius

= 2 x 15.51

= 31.02 cm

3 How many spherical balls of 1 cm radius can be made from a sphere of 32 cm diameter.

No. of balls x volume of small sphere = Volume of bigger sphere

$$N \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$$

$$N \times \frac{4}{3} \times \pi \times 1^3 = \frac{4}{3} \times \pi \times r^3$$

$$N = 16 \times 16 \times 16$$

= 4096 balls

4 Three brass balls of diameters 3 cm, 4 cm and 5 cm are melted and make into one solid ball, if there is no wastage. Find the diameter of the solid ball.

1st ball d_1 = 3 cm,

r_1 = 1.5 cm

2nd ball d_2 = 4 cm,

r_2 = 2 cm

3rd ball d_3 = 5 cm,

r_1 = 2.5 cm

Diameter of new ball = ?

Volume of new ball = Volume of 3 spherical balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (1.5^3 + 2^3 + 2.5)^3$$

$$r^3 = 3.375 + 8 + 15.625$$

$$r^3 = 27$$

$$r = \sqrt[3]{27}$$

$r = \sqrt[3]{3x3x3}$

$r = 3$ cm

Diameter of the ball $= 2 \times r$

$= 2 \times 3$

$= 6$ cm

5 Calculate the number of spheres that can be made if a solid metal cylinder of radius 14 cm and height 21 cm is melted and recast into spheres, each of radius 3.5 cm.

Cylinder radius = 3 cm,

height = 21 cm

Sphere radius = 3.5 cm

No. of sphere = ?

No. of balls x Volume of sphere = Volume of cylinder

$$N \times \frac{4}{3}\pi r^3 = \pi r^2 h$$

$$N \times \frac{4}{3} \times 3.5^3 = \times 14^2 \times 21$$

$$N = \frac{14 \times 14 \times 21 \times 3}{4 \times 3.5 \times 3.5 \times 3.5}$$

$$= \frac{12348}{171.5}$$

$= 72$ balls.

6 Two spheres of 3 cm radius melted together and formed into single sphere. Find the radius of the new sphere.

Volume of each sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (3)^3$

Volume of 2 spheres $= 2 \times \frac{4}{3}\pi \times (3)^3$

$$= \frac{4}{3}\pi \times 54 \text{ cubic cm}$$

Let R = Radius of new sphere made

Volume of new sphere $= \frac{4}{3}\pi R^3$ cubic cm

Assuming no wastage in material, we can say

Volume of new sphere = Total volume of two spheres

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 54$$

Deleting $\frac{4}{3}\pi$ from both sides (because of similarity), we can write:

$R^3 = 54$

R = Cube root of 54

$= \sqrt[3]{54}$

= 3.780 cm

Radius of new sphere formed = 3.780 cm

7 Find the ratio of the total surface of all the smaller balls to that of the original one if a spherical lead ball is melted and made into smaller balls of one third the radius of the original one. (i) How many such balls can be made?

Let the radius of the original ball = R

The radius of the small ball = R/3

Volume of the original ball $= \frac{4}{3}\pi \cdot R^3$

And, Volume of the small ball $= \frac{4}{3}\pi \cdot (R/3)^3$

$$= \frac{4}{3} \times \frac{\pi \cdot R^3}{27}$$

No. of small balls $= \frac{4}{3}\pi \cdot R^3 \div \frac{4}{3}\pi \cdot \frac{R^3}{27}$

$= 27$

Now, surface of the original ball $= 4\pi \cdot R^2$

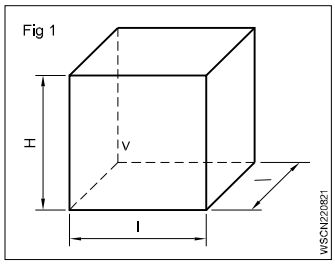
And, total surface of 27 small balls $= 27 \times 4\ \pi\ (R/3)^2$

Surface ratio $= 4\pi \cdot R^2 : 27.4\pi\ (R/3)^2$

$= 1:3$

Assignment

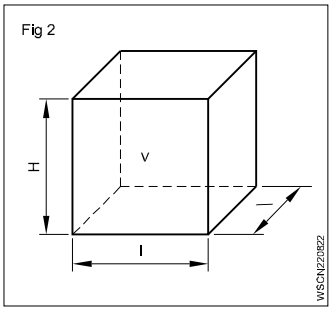
1



$l = 60$ mm

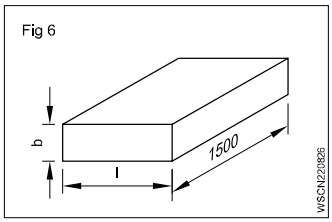
$V =$ _____ cm^3

2



$V = 5832\ cm^3$

$l =$ _____ mm

3 

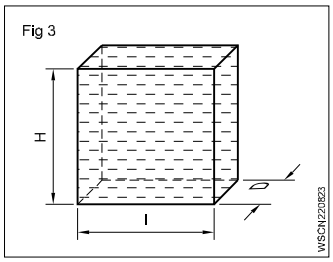
V = 1800 cm^3

$l : b$ = 3:1

H = 1500 mm

l = _____ mm

b = _____ mm

4 

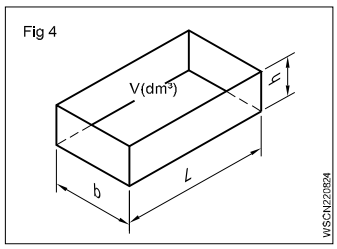
l = 1.5 metre

h = 0.8 metre

b = 0.45 metre

Capacity = _____ litres

1 m^3 = 1000 litres

5 

V = 140000 cm^3

l = 60 cm

b = 40 cm

H = _____ cm

Cube

1 Find the diagonal, lateral surface area, total surface area and volume of cube, whose side is 15 cm.

2 Find the volume of 10 cubes where each side is 5 cm.

3 Find its volume if a solid cube has each of its sides 60 mm long.

4 What is its side if the total surface area of a cube is 384 m^2.

5 Find out side of the cube if a cube has volume 422 cc.

Cuboid

1 Find the volume of the tank in m^3, if the length is 60 m, breadth 40 m and height 20 m.

2 Find the volume of a C.I. casting of a rectangular block having 25 cm x 20 cm x 8 cm size.

3 Calculate the total surface area of a box whose length, width and height are 120 cm, 50 cm and 60 cm respectively.

4 Find the volume of the sheet if a brass sheet is of 25 cm square and 0.4 cm thick.

5 Express its capacity in litres if a vessel measures 3m x 4m x 5m.

6 A milk tank with square base has a volume of holding 10 m^3 of milk. What will be the height of the tank, if its side is 2.583 m.

7 Find out its height if the cross section is 420 mm length and 230 mm wide rectangular and the capacity of the tank is 48 litres.

8 How many litres of water it can hold if a water tank has the following dimensions of Length = 4m, breadth = 3.2 m and height = 1400 cm?

Cylinder

1 Find the curved surface area of cylinder where
   a diameter 18 cm and height 34 cm
   b diameter 28 cm and height 42 cm

2 Find the total surface area of cylinder whose
   a diameter 24 cm and height 40 cm
   b diameter 42 cm and height 60cm
   c diameter 14 cm and height 35 cm

3 Find out the volume of cylinder whose
   a base is 10 cm radius and height is 40 cm
   b base is 7 cm radius and height is 12 cm
   c base is 35 cm diameter and height is 100 cm

4 Find the volume, C.S.A and T.S.A of a cylinder having diameter 10 cm and 20 cm height.

5 A cylindrical tank has 22000 cc water. If the depth of water is 70 cm. Calculate the diameter of the tank.

6 Find out its radius. if the volume of cylinder is 5544 cm^3 and height is 16 cm.

Sphere

1 Find the volume of sphere using the following dimension.
   a 3.5 cm diameter
   b 4 cm diameter
   c 7 cm diameter
   d 20 cm diameter
   e 5 cm diameter

2 Find the diameter of a sphere having volume of 512cc.

3 Find the total surface area of a sphere having
   a radius 1.75 cm.
   b radius 12 cm
   c radius 56 cm
   d diameter 20cm
   e radius 3 cm

4 How many spherical balls of 1 cm radius can be made from a sphere of 16 cm diameter.

5 Three balls of diameter 2m, 4cm and 6 cm are melted and made into one solid ball. If there is no wastage, find the diameter of solid ball.

6 How many solid spheres each radius 3 cm can be moulded from a solid metal cylinder whose length is 45 cm and base radius is 2 cm.

7 Calculate the number of balls made if a ball of 10 cm radius is to be converted into small ball of 2 cm radius.

Mensuration - Finding the lateral surface area, total surface area and capacity in litres of hexagonal, conical and cylindrical shaped vessels Exercise 1.8.43

Hexagonal bar

Volume of Hexagonal bar = Area of hexagonal x heigth

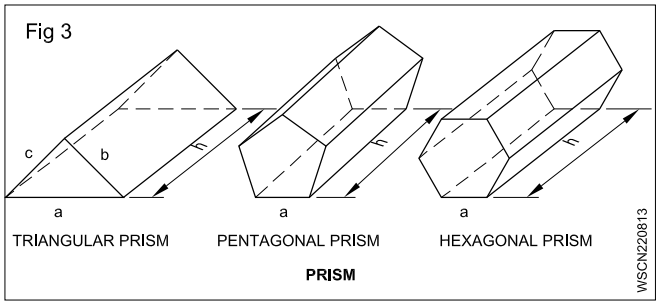
Lateral surface area of hexagonal bar

= 6 x length of the bar x side of hexagon

or = 3.464 x length of the bar x flat of hexagon

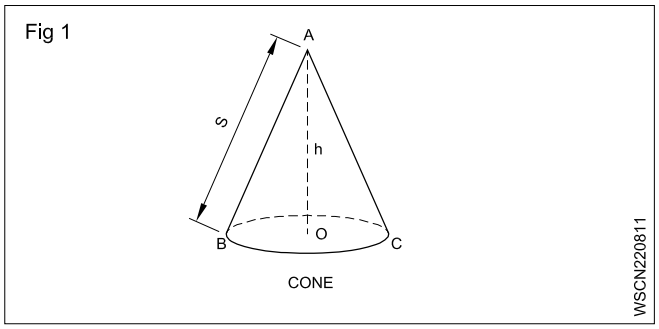
Total surface area of hexagonal bar

= lateral surface area + (2 x area of hexagon)



Cone

Cone is a pyramid with a circular base.



Volume of cone = $\frac{1}{3}\pi r^2 h$

or $= \frac{\pi}{12} d^2 h$

Curved area = πrs

Total surface area = $\pi r(s+r)$

Where r = radius of base

d = diametre of base

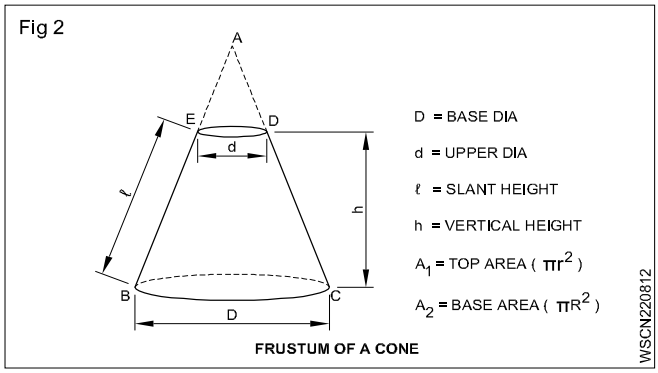
h = vertical height of cone

s = slant height $\sqrt{r^2+h^2}$

Frustum of a cone

When a cone is cut by a plane parallel to the base, and upper part is removed, the formation appears, is termed as frustum of a cone. Buckets, oil cans etc.are such frustums in shape.

L.S.A = π l (R + r) unit^2



TSA = π l (R + r) + A_1 + A_2 unit^2

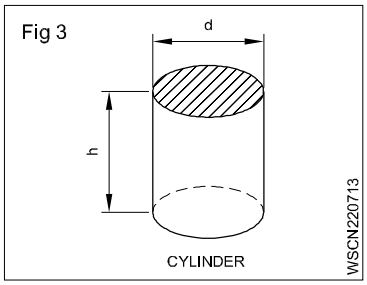
$V = \frac{\pi}{3} h (R^2 + Rr + r^2)$ unit^3

[A_1 = Top aera ; A_2 = Bottom area]

Cylinder

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$



Curved area of cylinder = $2\pi rh$

Total surface area of cylinder = $2\pi r(h+r)$

r = Radius of base

d = Diameter of base

h = Height of cylinder

Hollow cylinder

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.

Volume of hollow cylinder = $\pi (R^2 - r^2) h$ (or)

$= \pi (R + r) (R - r) h$ (or)

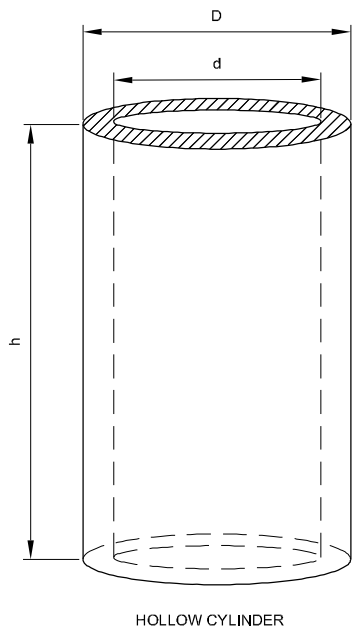
$= \frac{\pi}{4} (D^2 - d^2) h$

$= \frac{\pi}{4}(D+d)(D-d)h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

Fig 4



HOLLOW CYLINDER

$\therefore$ TSA : $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

Mean dia = $\frac{D-d}{2}$

If thickness given then:

Volume of hollow cylinder = π x mean dia x thickness x height

Example

1 Find the volume of an hexagonal prism having its side 20 cm and height 200 cm.

Side of hexagonal prism (a) = 20 cm

Height (h) = 200 cm

Volume (V) = Base side area x Height

$= 6 \times \frac{\sqrt{3}}{4} \times a^2 \times h$

$= 6 \times \frac{\sqrt{3}}{4} \times 20 \times 20 \times 200$

$= 1,20,000 \times \sqrt{3}$

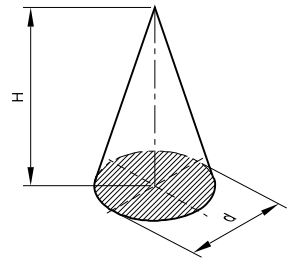
$= 1,20,000 \times 1.732$

$= \mathbf{2,07,840\ cm^3}$

Volume of the hexagonal prism = 2,07,840 cm^3

2 Calculate the height. Also find the lateral surface area if a cone has a base diameter of 210 mm and its volume is 3056 cm3.

Fig 6



Volume of a cone = $\frac{1}{3}$ x Area of base x height

$3056\ cm^3 = \frac{1}{3} \times 0.785 \times 210^2 mm^2 \times H$

$H = \frac{3056 \times 3 \times 1000 mm^3}{0.785 \times 210^2\ mm^2} = 264.82$ mm

L = Slant height = $\sqrt{264.83^2 + 105^2} = 284.9mm$

Lateral surface area = $\frac{1}{2}\ \pi$ X 210 X 284.9mm^2

= 94017 mm^2 = 940.17 cm^2

4 Determine its diameter in mm if the height of a rod of 1.6 metres and its volume is 1.017 $metre^3$.

V = A x H

$1.017\ metre^3 = 0.785d^2 \times 1.6$ metres

$0.785d^2 = \frac{1.017}{1.6} metre^2$

$d^2 = \frac{1.017}{1.6 \times 0.785} metre^2$

$= \frac{1.017}{1.6 \times 785} metre^2$

$d = \sqrt{\frac{10170}{16 \times 785}} metre$

$= \sqrt{\frac{10170}{12560}}$

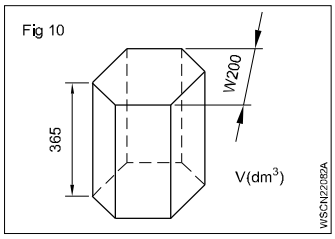
$= \sqrt{0.8097}$

$= 0.8998$

$= 899.8mm$

Assignment

1

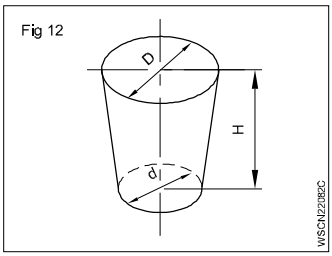


W = 200 mm

H = 365 mm

V = _____ mm^3

2



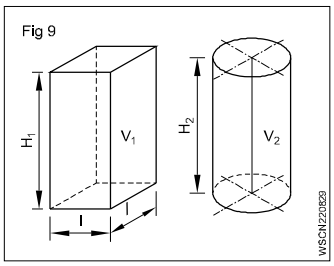
D = 290 mm

d = 180 mm

H = 320 mm

Capacity
= _____ litres

3



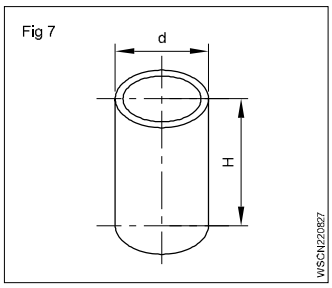
$V_1 = V_2$

$H_1 = H_2$

l = 35 cm

d = _____ cm

4

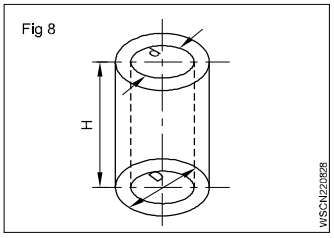


d = 35 cm

H = 450 mm

Capacity
= _____ litres

5



D = 175 mm

d = 115 mm

H = 420 mm

V = _____ mm^3