Introduction:

Trigonometry is the branch of mathematics which deals with the study of measurement and relationship of the three sides and three angles of a triangle.

Units:

Measurement of Angles

There are three systems of measuring the angle:

(i) Sexagesimal System

This is called British System. In this system, one right angle is divided into 90 equal parts which are called degrees.Each part is divided into 60 parts which are called minutes.Each minute is divided into 60 parts which are called seconds.The parts so divided respectively are called:

One degree (1°), one minute (1') and one second (1")

It means 1 right angle = 90° (90 degrees)

1 degree (1°) = 60' (60 minutes)

1 minute (1') = 60" (60 seconds)

In Trigonometry, mostly this system is used.

(ii) Centesimal System

This is called French System. In this system, the right angle is divided into 100 equal parts which are called grades. Each grade is divided into 100 minutes and each minute is divided into 100 seconds.

Parts so divided are respectively called:

One grade (1 g), one minute (1'), one second (1").

It means 1 right angle = 100 grades (100g)

1 grade (1 g) = 100 minutes (100')

1 minute (1') = 100 seconds (100")

90° = 100g (because each is a right angle)

This system is easier than Sexagesimal System. But to use this system many other systems will have to be devised that is why this system is not used.

(iii) Circular System

In this system, the unit of measuring angles is radian. It is that angle which is formed at the centre and is formed of an arc of length equal to radius in a circle.

There is one constant ratio between the circumference and dia of a circle. This is represented by π .

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant point} = \pi$$

Circumference = π x dia

= $2\pi r$ (where r is radius of the circle)

π =

Circumference makes an angle ($2\pi r$) = 360°

Radius of the circle makes an angle (r) = 1 Radian

$$\text{ie :} \quad \frac{C}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$\frac{2\pi r}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi = \frac{360^\circ}{1\text{Radian}}$$

2π Radian = 360°

π Radian = 180°

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180^\circ} \text{ Radian}$$

Examples

1 Convert 45°36'20" into degree and decimal of degree.

60 second = 1 minute

$$20 \text{ second} = \frac{20}{60} = 0.333'$$

60 minute = 1 degree

$$36.333 \text{ minute} = \frac{36.333}{60} = 0.606^\circ$$

$45^0 36'20" = 45.606^\circ$

2 Convert 24.59° into degree, minute and second

1 degree = 60 minute

0.59 degree = 0.59 x 60 = 35.4'

1 minute = 60 second

0.4 minute = 60 sec x 0.4

= 24"

Therefore $24.59^\circ = 24^0 35'24"$

3 Change $50^0 37'30"$ into degrees

By changing angle degrees into decimals

$$30" = \frac{30}{60} = 0.50'$$

37'30" = 37.5'

$$37.5' = \frac{37.5}{60} = 0.625^0$$

$50^0 37'30" = 50.625^0$

4 Convert $23^0 25' 32''$ into radians

We know $1^0 = 60' = 3600''$

Therefore $23^0 25' 32''$

$$= \left(23 + \frac{25}{60} + \frac{32}{3600}\right) degrees$$

$$= \frac{82800 + 1500 + 32}{3600}$$

$$= \frac{84332}{3600}$$

But $180^0 = \pi$ radians

Therefore 23.4255 dgrees

$$= \frac{23.4255}{180}\ \pi \text{ radians}$$

$$= \frac{23.4255}{180} \times \frac{22}{7} \text{ radians}$$

= 0.4089 radians

5 Convert $87^0 19' 57''$ into Radian.

$$19'57'' = 19' + \frac{57'}{60}$$

$$= 19' + 0.95'$$

$$= 19.95'$$

$$87°19.95' = 87° + \frac{19.95^{\circ}}{60}$$

$$= 87° + 0.332° = 87.33°$$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$87.33° = \frac{\pi}{180} \times 87.33 \text{ radian}$$

$$= 1.524 \text{ radian}$$

6 Convert $67°11'43''$ into Radian

$$11'43'' = 11' + \frac{43'}{60}$$

$$= 11' + 0.716'$$

$$= 11.72'$$

$$67°11.72' = 67° + \frac{11.72^{\circ}}{60}$$

$$= 67° + 0.195°$$

$$= 67.2°$$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$67.2° = \frac{\pi}{180} \times 67.2 \text{ radian}$$

$$= 1.173 \text{ radian}$$

7 Convert $\frac{4}{7}\pi$ radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{4}{7}\pi \text{ radian} = \frac{180}{\pi} \times \frac{4}{7}\pi \text{ degree}$$

$$= 102.9 \text{ degree}$$

$$= 102^0\ 0.9 \times 60'$$

$$= 102^0\ 54'$$

8 Convert 0.8357 radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$0.8357 \text{ radian} = \frac{180}{\pi} \times 0.8357 \text{ degree}$$

$$= 47.88^0$$

$$= 47^0\ 0.88 \times 60'$$

$$= 47^0\ 52.80'$$

$$= 47^0\ 52'0.8 \times 60''$$

$$= 47^0\ 52'48''$$

9 Convert 2.752 radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$2.7520 \text{ radian} = \frac{180}{\pi} \times 2.752 \text{ degree}$$

$$= 157.7^0$$

$$= 157.7^0 \times 60'$$

$$= 157^0 42'$$

10 Convent $\frac{3}{5}\pi$ radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{3}{5}\pi \text{ radian} = \frac{180}{\pi} \times \frac{3}{5}\pi \text{ degree}$$

$$= 108^0$$

Assignment

Convert into Degree

1. 12 Radian

Convert into Radians

2. 78^o
3. $47^0 20'$
4. $52^0 36' 45''$
5. $25^0 38''$

Convert into degree, minute and seconds

6. 46.723^o
7. 68.625^o
8. 0.1269 Radians
9. 2.625 Radians
10. 3/5 Radians

Dependency

The sides of a triangle bear constant ratios for a given definite value of the angle. That is, increase or decrease in the length of the sides will not affect the ratio between them unless the angle is changed. These ratios are trigonometrical ratios. For the given values of the angle a value of the ratios $\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}$ and $\frac{AC}{BC}$ do not change even when the sides AB, BC, AC are increased to AB', BC' and AC' or decreased to AB", BC" and AC".

For the angle

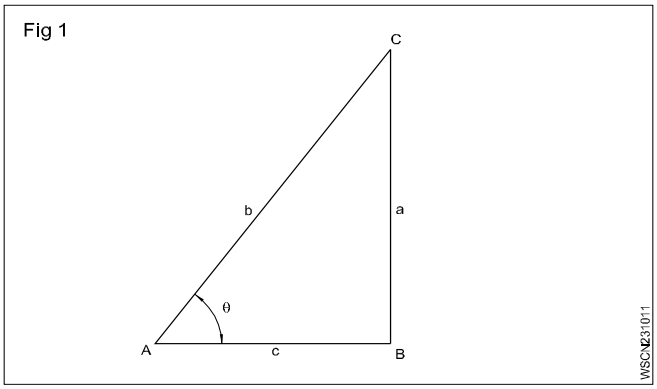
AC is the hypotenuse

AB is the adjacent side

BC is the opposite side.

The ratios

Fig 1



The six ratios between the sides have precise definitions.

$$\text{Sine } \theta = \frac{BC}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \text{Sin } \theta$$

$$\text{Cosine } \theta = \frac{AB}{AC} = \frac{\text{Adjecent side}}{\text{Hypotenuse}} = \text{Cos } \theta$$

$$\text{Tangent } \theta = \frac{BC}{AB} = \frac{\text{Opposite side}}{\text{Adjecent side}} = \text{Tan } \theta$$

$$\text{Cosecant } \theta = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \text{Cosec } \theta$$

$$\text{Secant } \theta = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Adjecent side}} = \text{Sec } \theta$$

$$\text{Cotangent } \theta = \frac{AB}{BC} = \frac{\text{Adjecent side}}{\text{Opposite side}} = \text{Cot } \theta$$

Relationship between the ratios

$$\text{Cosec } \theta = \frac{AC}{BC} = \frac{1}{\frac{BC}{AC}} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{AC}{AB} = \frac{1}{\frac{AB}{AC}} = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{AB}{BC} = \frac{1}{\frac{BC}{AB}} = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{\text{side}BC}{\text{side}AC} = \frac{a}{b}$$

$$\cos\theta = \frac{\text{side } AB}{\text{side}AC} = \frac{c}{b}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$$

$$= \frac{\text{side } BC}{\text{side } AB} = \tan\theta$$

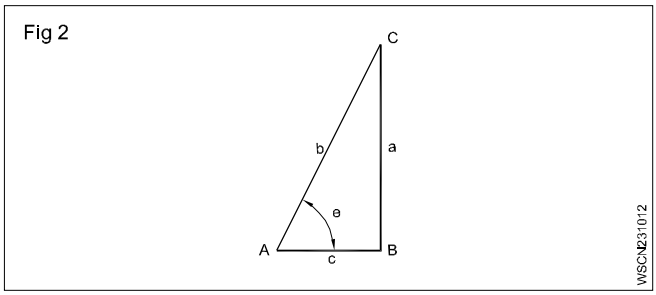
$$\sin\theta = \frac{1}{\text{cosec } \theta} \text{ or cosec } \theta = \frac{1}{\sin\theta} \text{ or } \sin\theta.\text{cosec } \theta = 1$$

$$\cos\theta = \frac{1}{\sec\theta} \text{ or sec } \theta = \frac{1}{\cos\theta} \text{ or cos } \theta . \sec\theta = 1$$

$$\tan\theta = \frac{1}{\cot\theta} \text{ or } \cot\theta = \frac{1}{\tan\theta} \text{ or cot } \theta . \tan\theta = 1$$

By pythogoras theorem we have, $AC^2 = AB^2 + BC^2$

Fig 2



Dividing both sides of the eθuation by AC^2, we have

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \left[\frac{AB}{AC}\right]^2 + \left[\frac{BC}{AC}\right]^2$$

$$1 = (\cos\theta)^2 + (\sin\theta)^2$$

$$\sin^2\theta + \cos^2\theta = 1$$

Sine, Cosine, Tangent, Cosec, Sec and Cotangent are the six trigonometrical ratios

$\tan\theta = \frac{\text{Sin } \theta}{\text{Cos } \theta}$ and $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \cos^2\theta = 1$$

It can be transformed as

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\text{or } \cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

We know $\sin^2\theta + \cos^2\theta = 1$

Dividing both sides by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\text{or } 1 + \tan^2\theta = \sec^2\theta$$

Using the same eθuation

$$\sin^2\theta + \cos^2\theta = 1.$$

Dividing both sides by $\sin^2\theta$,

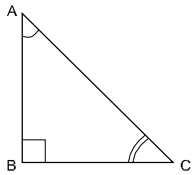
$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Relation between the trignometrical ratio (Fig 3)

Fig 3



WSCN231013

$$\sin A = \frac{BC}{AC}$$

$$\cos A = \frac{AB}{AC}$$

$$\tan A = \frac{BC}{AB}$$

$$\sin C = \frac{AB}{AC}$$

$$\cos C = \frac{BC}{AC}$$

$$\tan C = \frac{AB}{BC}$$

$$\angle A + \angle B + \angle C = 180^0$$

$$A = 90^0 - \angle C$$

$$C = 90^0 - \angle A$$

$$B = 90^0$$

$$\sin A = \frac{BC}{AC} = \text{Cos}C$$

sinA = cosC = cos (90 - A)

$$\text{Sin } \theta = \cos(90 - \theta)$$

$$\cos A = \frac{AB}{BC} = \text{Sin } C$$

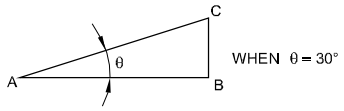
Cos A = Sin C = Sin (90- A)

$$\text{Cos } \theta = \text{Sin } 90 - \theta$$

The values of the trigonometrical ratios

When θ = 0° (Fig 4)

Fig 4



WSCN231014

When θ comes closer and closer to 0°, point C approaches B closer and closer and when **θ** = 0°, point C coincides on B so that BC = 0 and AB = AC.

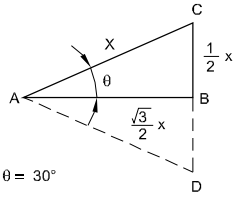
$$\sin 0° = \frac{BC}{AC} = 0$$

$$\cos 0° = \frac{AB}{AC} = 1$$

$$\tan 0° = \frac{BC}{AB} = \frac{0}{AB} = 0$$

When θ = 30° (Fig 5)

Fig 5



WSCN231015

CB is extended to D to make BD = CB. AD is joined. The two right angled triangles ACB and ADB are congruent.

AC = AD. The triangle ACD is an eθuilateral triangle.

If AC = x, CB = $\frac{x}{2}$

Then AB = $\frac{\sqrt{3}}{2}x$

$\sin\theta = \frac{CB}{AC} = \frac{\frac{1}{2}x}{x} = \frac{1}{2}$

$\cos\theta = \frac{AB}{AC} = \frac{\frac{\sqrt{3}}{2}x}{x} = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{CB}{AB} = \frac{\frac{1}{2}x}{\frac{\sqrt{3}}{2}x} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

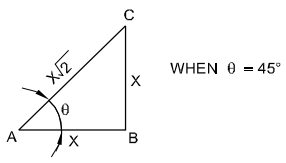
AC = x

CB = $\frac{1}{2}x$

AB = $\frac{\sqrt{3}}{2}x$

When θ = 45° (Fig 6)

Fig 6



WSCN231016

∠CAB = 45° ∠ACB = 45°

Triangle ACB is a right angled isosceles triangle.

side AB = BC . Let it be x .

Then AC the hypotenuse = $\sqrt{2}x$

$\sin\theta = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$.

$\cos\theta = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

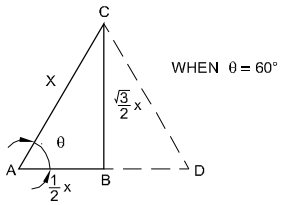
$\tan\theta = \frac{x}{x} = 1$.

AB = BC = x

AC = $\sqrt{2}x$

When θ = 60° (Fig 7)

Fig 7



WSCN231017

Extend AB to D such that BD = AB. Join CD . The two triangles ACB and DCB are congruent.

side AC = side DC. The triangle ACD becomes an eθuilateral triangle.

Let side AC = x ,

Then AB = $\frac{1}{2}x$, and BC = $\frac{\sqrt{3}}{2}x$

$\sin\theta = \frac{BC}{AC} = \frac{\frac{\sqrt{3}}{2}x}{x} = \frac{\sqrt{3}}{2}$

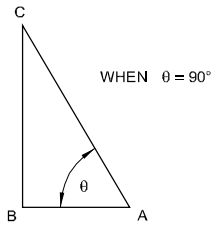
$\cos\theta = \frac{AB}{AC} = \frac{\frac{1}{2}x}{x} = \frac{1}{2}$

$\tan\theta = \frac{BC}{AB} = \frac{\frac{\sqrt{3}}{2}x}{\frac{1}{2}x}$

$= \frac{\sqrt{3}}{2} \times \frac{2}{1} = \frac{\sqrt{3}}{1} = \sqrt{3}$.

When θ = 90° (Fig 8)

Fig 8



WSCN231018

When θ becomes closer and closer to 90°, point A goes closer and closer to B and when θ = 90° point A coincides with B , making AC = BC and AB = 0.

$\sin\theta = \frac{BC}{AC} = 1$

$\cos\theta = \frac{AB}{AC} = \frac{0}{AC} = 0$

$\tan\theta = \frac{BC}{AB} = \frac{BC}{0} = \alpha$

Ratio	0°	30°	45°	60°	90°
sin θ	0		$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	¥

When θ increases,

Sine value increases;

Cosine value decreases;

Tangent value increases to more than 1 when the angle is more than 45° (tan60° = 1.732)

Sine of an angle = Cosine of its complementary angle

Cosine of an angle = Sine of its complementary angle

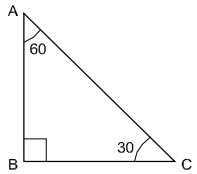
Examples

If $\sin 30^0 = \frac{1}{2}$ find the value of $\sin 60^0$

By applying pythagores theorem

$BC^2 = AC^2 - AB^2$

Fig 9



WSCN231019

$BC^2 = 2^2 - 1^2$

$= 4 - 1$

$= 3$

$BC = \sqrt{3}$

$\sin 60^0 = \frac{\sqrt{3}}{2}$

$\text{Cos}\theta = \frac{3}{5}$ Find the other trigonometrical ratios

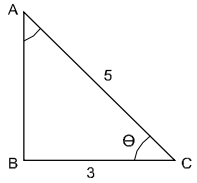
By applying pythagores theorem

$AB^2 = AC^2 - BC^2$

$= 5^2 - 3^2 \quad = 25 - 9$

$= 16$

Fig 10



WSCN231101A

$AB = \sqrt{16} = 4$

Now $\sin\theta = \frac{4}{5}$

$\tan\theta = \frac{4}{3}$

$\text{Cosec } \theta = \frac{5}{4}$

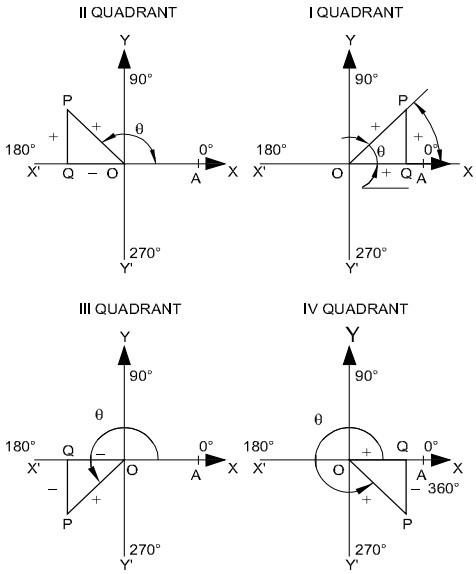
$\sec\theta = \frac{5}{3}$

$\cot\theta = \frac{3}{4}$

Angles and ratios at four quadrants

1st quadrant (0° to 90°) (Figs 11 & 12)

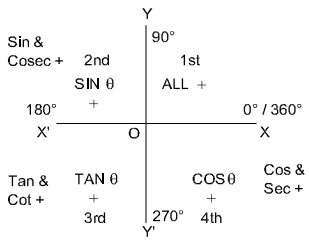
Fig 11



ANGLES AND RATIOS AT 4 QUADRANTS

WSCN23101B

Fig 12



POSITIVE TRIGNOMETRIC RATIOS OF 4 QUADRANTS
(REMAINING RATIOS –NAGATIVE)

WSCN23101C

$\frac{+PQ}{+OP} = +\sin\theta$

$\frac{+OQ}{+OP} = +\cos\theta$

$\frac{+PQ}{+OQ} = +\tan\theta$

2nd quadrant (More than 90° & less than 180°)

$\sin\theta = \sin(180^\circ - \theta)$

$= \frac{+PQ}{+OP} = +\sin(180^\circ - \theta)$

Therefore, $\sin\theta = +\sin(180^\circ - \theta)$

$\cos\theta = \cos(180^\circ - \theta)$

$= \frac{-OQ}{+OP} = -\cos(180^\circ - \theta)$

Therefore, $\cos\theta = -\cos(180^\circ - \theta)$

$\tan\theta = \tan(180^\circ - \theta)$

$= \frac{+QP}{-OQ} = -\tan(180^\circ - \theta)$

Therefore, $\tan\theta = -\tan(180^\circ - \theta)$

3rd quadrant (More than 180° & less than 270°)

$\text{Sin}\,\theta = \sin(\theta - 180^\circ)$

$= \frac{-QP}{+OP} = -\sin(\theta - 180^\circ)$

Therefore, $\sin\theta = -\sin(\theta - 180^\circ)$

$\text{Cos}\,\theta = \cos(\theta - 180)$

$= \frac{-OQ}{+OP} = -\cos(\theta - 180)$

Therefore, $\cos\theta = -\cos(\Theta - 180^\circ)$

$\tan\theta = \tan(\Theta - 180^\circ)$

$= \frac{-QP}{-OQ} = +\tan(\theta - 180^\circ)$

Therefore, $\tan\theta = +\tan(\theta - 180^\circ)$

4th quadrant (More than 270° & less than 360°)

$\text{Sin}\,\theta = \sin(360^\circ - \theta)$

$= \frac{-QP}{+OP} = -\sin(360^\circ - \theta)$

Therefore, $\sin\theta = -\sin(360^\circ - \theta)$.

$\text{Cos}\,\theta = \cos(360^\circ - \theta)$

$= \frac{+OQ}{+OP} = +\cos(360^\circ - \theta)$

Therefore, $\cos\theta = +\cos(360^\circ - \theta)$.

$\tan\theta = \tan(360^\circ - \theta)$

$= \frac{-QP}{+OQ} = -\tan(360^\circ - \theta)$

Therefore, $\tan\theta = -\tan(360^\circ - \theta)$.

Signs of trigonometrical functions for angles more than 90^0

Ratio	90 - θ	90 + θ	180 - θ	180 + θ	270 - θ	270 + θ	360 - θ	- θ
sin	cos	cos	sin	- sin	- cos	- cos	- sin	- sin
cos	sin	- sin	- cos	- cos	- sin	sin	cos	cos
tan	cot	- cot	- tan	tan	cot	- cot	- tan	- tan
cosec	sec	sec	cosec	- cosec	- sec	- sec	- cosec	- cosec
sec	cosec	- cosec	- sec	- sec	- cosec	cosec	sec	sec
cot	tan	- tan	- cot	cot	tan	- tan	- cot	- cot

Simplify :

$\cot\theta + \tan(180+\theta) + \tan(90-\theta) + (\tan 360 - \theta)$

$= \cot\theta + \tan\theta - \cot\theta - \tan\theta$

$= 0$

Simplify :

$$\frac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cos(90-\theta)}$$

$$= \frac{(-\sin\theta)\,x\,(\sec\theta)\,x\,(-\tan\theta)}{(\sec\theta)\,x\,(-\sin\theta)\,x\,(-\sin\theta)}$$

$$= \frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta} = \sec\theta$$

simplify:

$$\frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$$

$\cos(90^0 + \theta) = -\sin\theta$

$\sec(-\theta) = \sec\theta$

$\tan(180^0 - \theta) = -\tan\theta$

$\sec(360^0 - \theta) = \sec\theta$

$\sin(180^0 + \theta) = -\sin\theta$

$\cot(90^0 + \theta) = -\tan\theta$

$$\frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$$

$$= \frac{(-\sin\theta)(\sec\theta)(\tan\theta)}{(\sec\theta)(-\sin\theta)(-\tan\theta)}$$

$= 1$

Simplify:

$\cot\theta + \tan(180^0 + \theta) + \tan(90^0 + \theta) + \tan(360^0 - \theta)$

$\tan(180^0 - \theta) = \tan\theta$

$\tan(90^0 + \theta) = -\cot\theta$

$\tan(360^0 - \theta) = -\tan\theta$

$\cot\theta + \tan(180^0 + \theta) + \tan(90^0 + \theta) + \tan(360^0 - \theta)$

$\cot\theta + \tan\theta - \cot\theta - \tan\theta = 0$

Assignment

1 Given $\sin 30^0 = 1/2$, find the value of $\tan 60^0$

2 If $\cos\theta = 4/5$, find the other radios

3 If sin A = 3/5, find $\cos\theta$, $\tan\theta$ & $\sec\theta$

4 If $\tan\theta = 24/7$, find $\sin\theta$ and $\cos\theta$

5 Find the value of $\cos\theta$ and $\tan\theta$, if $\sin\theta = 1/2$

6 If $\cos\theta = 5/13$, find the value of $\tan\theta$

7 If $\sin\theta = 1/2$, find the value of $\sin^2\theta - \cos^2\theta$

8 i What is the value of $\cos\theta$ and $\tan\theta$

$$\sin\theta = \frac{4}{5}$$

ii What is the value of $\sin\theta$ and $\cos\theta$

$$\tan\theta = \frac{12}{5}$$

9 What is the value of

$$\frac{\sin^2 30^\circ}{\cos^2 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Simplify :

1 $\tan(90 + A) + (\tan 180 + A)\tan(90 + A)$

2 $$\frac{\cos(90+\theta)\cdot\sec(-\theta)\cdot\tan(180-\theta)}{\sec(360+\theta)\cdot\sin(180+\theta)\cdot\cot(90+\theta)}$$

What is the value of

3 $\sin 160^0$

4 $\sin 450^0$

5 $\cos 135^0$

6 $\tan 260^0$

Use of trigonometrical tables

	Minutes from 0 to 4						Mean difference				
Deg.	0'	6'	12'	18'	24'	54'	1'	2'	3'	4'	5'
0				.				.			
1				.				.			
2				.				.			
3				.				.			
.				.				.			
.				.				.			
.				.				.			
26	..	..	..	x	..	..	..	5			
.											
.											
.											
89											

Sine value for 26°– 20'

Refer to Natural sine table.

Degrees column go up to 26° down

Minutes column 18' horizontal and under this note the value which is given as 0.4431.

Under mean difference for 2' in the same horizontal line 5 is given. Add this to the extreme right number noted for 26°– 18'.

Sine 26° – 20' = 0.4431 + .0005 = 0.4436

Cosine value for 43° – 41'

Referring to the Natural cosines table for 43° –36' it is given as 0.7242 and the mean difference for 5' minutes is given as 10.

cos 43°– 41' = value for cos 43°.36 '

– the value given for

mean difference of 5 ' = 0.7242 – 0.0010

= 0.7232

When reading sine value add the mean difference value. When reading cosine value subtract the mean difference value.

Arrangement

Values of trigonometrical ratios can be taken from mathematical tables.

The left hand vertical column consists of degrees.

The top horizontal column is arranged in minutes in steps of 6' from 0' to 54'. In the extreme right horizontal columns the mean differences are written in minutes from 1' to 5' in steps of 1' to account for angles with minutes between the interval of 6'.

- **The values of cosine, cosecant and cotan gent decrease when the value of the angle increases.**
- **For sine, secant and tangent, the value increases when the angle increases.**
- **The value of sine and cosine will never be more than 1.**
- **The value of secant and cosecant will never be less than 1.**
- **The value of Tan and Cot ranges from 0 to ∞.**

EXAMPLE

From the tables obtain the cosine of 45°–20 '.

cos 45°–18 ' = 0.7108

mean difference for 2 ' = 0.0004

cos 45°– 20 ' = 0.7104

— — — — — — —

SINE TABLE

1 Sin 25^0 = 0.4226

2 Sin 17^0 5'

$\sin 17^0$	=	0.2924
Difference 5'	=	14
$\sin 17^0$ 5'	=	0.2938 Ans

$\sin 17^0$ 45' 13"

$\sin 17^0$ 46'	=	0.3051
$\sin 17^0$ 45'	=	0.3048
Difference 1'	=	0.0003

$1'(\text{or})\,60'' = 0.0003$

$13'' = \frac{0.0003}{60} \times 13$

$= \frac{0.0039}{60}$

$= \frac{0.00039}{6}$

$= 0.000065$

$\sin 17^0\,45' = 0.3048$

$13'' = 0.000065$

$\sin 17^0\,45'13'' = 0.304865$ Ans.

4 $\sin 82^\circ\ 14'18''$

$\sin 82^\circ\ 15' = 0.9908$

$\sin 82^\circ\ 14' = 0.9908$

Difference $1' = 0$

$1'(\text{or})\,60'' = 0$

$\sin 18'' = 0$

$\sin 82^\circ\ 14' = 0.9908$

$18'' = 0.0000$

$\sin 82^\circ 41'18'' = 0.9908$ Ans.

Finding the corresponding angles when sine values are given:

1. Sin θ = 0.9925

θ = 83⁰

2. Sin θ = 0.8791

$0.8788 = \text{Sin } 61^0\,30'$

$0.0003 = 2'$

$0.8791 = \text{Sin } 61^0\,32'$

3. sin θ = 0.68015

$0.6794 = \sin 42^0\,48'$

$0.0006 = 3'$

$0.6800 = \sin 42^0\,51'$

$0.6803 = \sin 42^0\,52'$

Difference $0.0003 = 1'$ (or) $60''$

$0.00015 = \frac{60}{0.0003} \times 0.00015$

$= \frac{60 \times 15}{30}$

$= 30''$

$0.6800 = \sin 42^0\,51'$

$0.00015 = 30''$

$0.68015 = \sin 42^0\,51'30''$

$\theta = 42^0\,51'30''$

4. sin θ = 0.84756

$0.8471 = \sin 57^0\,54'$

$0.0003 = 2'$

$0.8474 = \sin 57^0\,56'$

$0.8476 = \sin 57^0\,57'$

Difference $0.0002 = 1'$ (or) $60''$

$0.00016 = \frac{60}{0.0002} \times 0.00016$

$= \frac{60 \times 16}{20}$

$= 48''$

$0.8474 = \sin 57^0\,56'$

$0.00016 = 48''$

$0.84756 = \sin 57^0\,56'48''$

$\theta = 57^0\,56'48''$

5. sin θ = 0.6

$0.5990 = \sin 36^\circ 48'$

$0.0009 = 4'$

$0.5999 = \sin 36^\circ\,52'$

$0.6002 = \sin 36^\circ\,53'$

difference $0.0003 = 1'$ (or) $60'$

$0.0001 = \frac{60}{0.0003} \times 0.0001 = \frac{60}{3} \times 1$

$0.5999 = \sin 36^\circ 52'$

$0.0001 = 20''$

$0.6000 = \sin 36^\circ 52'20''$

$\theta = \mathbf{36^\circ\ 52'\ 20''}$

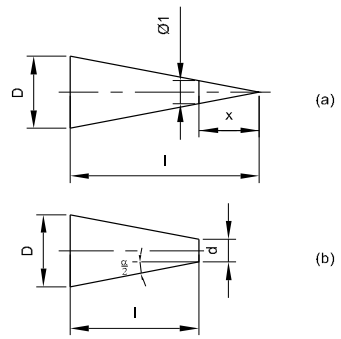
Calculations involving tapers

D - Big diameter of the taper

d - small diameter of the taper

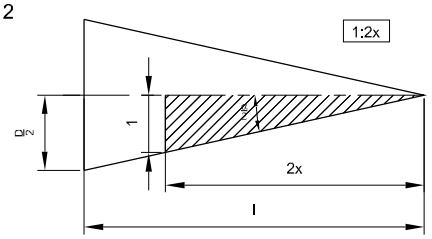
C - Taper Ratio - 1:x

Fig 1



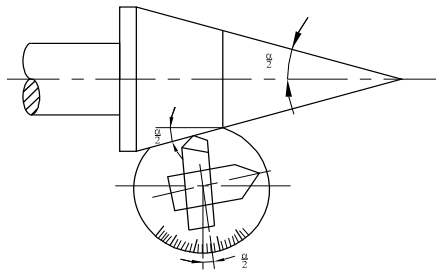
WSCN231111

Fig 2



WSCN231112

Fig 3



WSCN231113

$\frac{C}{2}$ Ratio of inclination - 1:2 **x**

l - length of taper

a - included angle of taper

$\frac{\alpha}{2}$ - setting angle

Taper ratio = Ratio of inclination (for wedges).

Taper ratio

The ratio between the difference in diameter to the length of the taper is known as taper ratio. *D* is the difference in larger diameter shown in the sketch as the small diameter of taper is 0. Taper ratio is *D : l* . In the sectioned portion the difference in diameter is 1 and the length of taper is shown as *x* .

C = D: d = 1 : x as per Fig 1 (a), C = $\frac{D-d}{l}$ as per Fig 1 (b)

Ratio of inclination

Taking half of the taper, $\frac{D}{2}$ is the difference in diameter for a taper length of l, if d = 0.

$\therefore \frac{C}{2} = \frac{D}{2l}$ if the small diameter is 0

or $\frac{C}{2} = \frac{D-d}{2l}$

1 Ratio of inclination = $\frac{1}{2}$ of the taper ratio.

Setting angle

One of the methods of turning taper is by swivelling the compound slide to an angle known as setting angle and feeding the tool at an angle to the axis of work.

$$\tan\frac{\alpha}{2} = \frac{C}{2} = \frac{D-d}{2l}$$

$$\tan\frac{\alpha}{2} = \frac{\text{taper ratio}}{2}$$

$$= \frac{\text{difference in diameter}}{2 \text{ x taper length}}$$

NOTE: $\frac{\alpha}{2}$ is the setting angle which is eθual to half of the included angle of the taper.

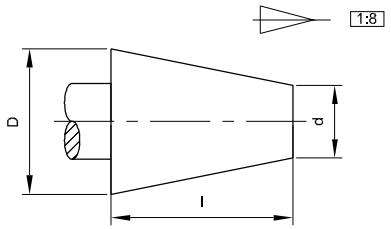
Taper Ratio *C* = 1 : *x* or *D* : *l* or *(D –d)* : *l*

Ratio of inclination $\frac{C}{2} = 1:2x = \frac{D}{2} : l$ or $\left(\frac{D-d}{2}\right)$:l

Setting angle determination is by the formula

$$\tan\frac{\alpha}{2} = \frac{D-d}{2l} = \frac{C}{2}$$

Fig 4



WSCN231114

EXAMPLE

A pivot in the form of a frustum of a cone has a taper ratio 1:8. If the small diameter is 30 mm and length of taper is 80 mm, find its large diameter.

C = 1:8 = $\frac{1}{8}$

$\therefore \frac{D-d}{l} = \frac{1}{8}$

$\therefore D - d = \frac{1}{8} = \frac{80}{8} = 10\,\text{mm}.$

D – 30 mm = 10 mm

D = 10 mm + 30 mm = 40 mm

Large diameter *D* = 40 mm

Cos Table

1. Cos 38°

Cos 38° = 0.7880

2. Cos 83° 12'

Cos 83°12' = 0.1184

3. Cos 26° 40'

Cos 26°36' = 0.8942

4' = 5 (-)

Cos 26°40' = 0.8937

---

4. Cos 31° 20'

Cos 31°18' = 0.8545

2' = 3 (-)

Cos 31°20' = 0.8542

Find the corresponding angles when cos values are given:

1. Cos θ = 0.5150

θ = 59°

2. Cos θ = 0.0192

θ = 88°54'

3. Cos θ = 0.9682

0.9686 = cos 14°24'

(-) 4 = 5'

0.9682 = cos 14° 29'

θ = 14° 29'

4. Cos θ = 0.8476

0.8480 = cos 32° 0'

(-) 0.0003 = 2'

0.8477 = cos 32° 2'

0.8475 = cos 32° 3'

0.0002 = 1' (or) 60"

$$0.0001 = \frac{60}{0.0002} \times 0.0001$$

$$= \frac{60}{2} \times 1$$

= 30"

0.8477 = Cos 32° 2'

(-) 0.0001 = 30" (+)

0.8476 = Cos 32° 2' 30"

---

tan Table

1. tan 35° 37'

tan 35° 36' = 0.7159

1' = 0.0004

---

tan 35° 37' = 0.7163

---

2. tan 50° 5'

Tan 50° 0' = 1.1918

5' = 0.0036

Tan 50° 5' = 1.1954

Find the corresponding angles when tan values are given

1. tan θ = 0.3972

0.3959 = tan 21° 36'

0.0013 = 4'

0.3972 = tan 21° 40'

---

2. tan θ = 1.0065

1.0035 = tan 45° 6'

0.0030 = 5'

1.0065 = tan 45° 11'

Problems Related with Trigonometrical tables

1. A 250 mm Sine bar is used to measure an angle. If the difference in height is 5 cm, find the angle.

$$Sin\theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$$= \frac{5 \text{ cm}}{250 \text{ mm}}$$

$$= \frac{50 \text{ mm}}{250 \text{ mm}}$$

= 0.2000

θ = 11° 32'

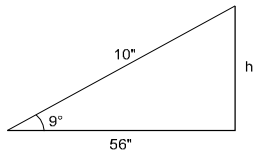
2 Find the height of the slip gauge if a Sine bar with plugs of 10" centre is set up to inspect a taper having an included angle of 9° 56".

$$Sin\theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$$\text{Sin } 9°56" = \frac{h}{10}$$

$\therefore h = 10 \times \sin 9°56"$

Fig 5



WSCN231115

Sin 9° 0" = 0.1564

Sin 9° 1" = 0.1567

1' (or) 60" = 0.0003

$$56" = \frac{0.0003}{60} \times 56$$

$$= \frac{0.0168}{60}$$

$$= \frac{0.00168}{6}$$

$$= 0.00028$$

sin9° 0' 00" = 0.1564

56" = 0.00028

---

sin9° 0' 56" = 0.15668

---

h = 10 x sin9° 56"

= 10 x 0.15668

= 1.5668 cm

Height of slip gauge = 1.5668"

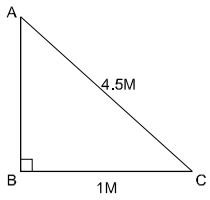
3 Find the angle which the ladder makes with the ground if the foot of a 4.5 m long ladder is placed at 1 m away from the wall.

In Right angled D

$$\text{Cos C} = \frac{BC}{AC}$$

$$\text{Cos } \theta = \frac{1\text{ m}}{4.5\text{ m}} = 0.2222$$

Fig 6



WSCN231116

Cos = 0.2222

0.2233 = cos 77° 6'

(-) 0.0011 = 4' (+)

0.2222 = cos 77° 10'

θ = 77°10'

Assignment

I Find the values of the given angles

1 Sin 65°

2 Sin 42°23'

3 Sin 66° 35' 32"

4 Sin 7° 15' 41"

5 Sin 27° 27"

6 Cos 47° 39'

7 Cos 47° 39'

8 Cos 79° 31' 53"

9 Tan 28° 45'

10 Tan 67° 27' 36"

II Find corrosponding angles for given values

1 Sin θ = 0.3062

2 Sin θ = 0.6002

3 Sin θ = 0.22453

4 Sin θ = 0.04802

5 Cos θ = 0.6446

6 Cos θ = 0.8926

7 Cos θ = 0.11773

8 Cos θ = 0.21646

9 Tan θ = 0.3411

10 Tan θ = 2.3868

III

1 Calculate its base. if the slant height of a cone is 12.25 cm and the vertex angle is 110^0.

2 A ladder 2.5 m long makes an angle of 60^0 with the ground. Find the height of the wall where the ladder touches the wall.

3 A sine bar of 200 mm is to be set at an angle of $15^0 15'3"$. Select the slip gauge block to built up the reθuired height.

4 In a right angled triangle ABC, ∠C = 90°, If AB = 50 mm and ∠B = 75°, Find the remaining sides.

5 Calculate the reθuired length of the bar for this point if a centre point having an included angle of 60° is to be turned at the end of a 50 mm dia bar.

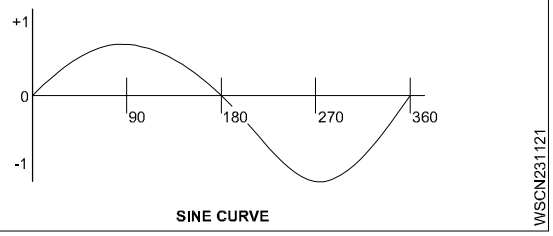
Natural Sines

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	3	6	9	12	15
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	3	6	9	12	15
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	3	6	9	12	14
5	0.0872	0.0899	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	3	6	9	12	14
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	3	6	9	12	14
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	3	6	9	12	14
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	3	6	9	11	14
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	3	6	9	11	14
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	3	6	9	11	14
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	3	6	9	12	14
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	3	6	9	11	14
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	3	6	8	11	14
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	3	6	8	11	14
15	0.2558	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	3	6	8	11	14
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	3	6	8	11	14
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	3	6	8	11	14
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	3	6	8	11	14
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	3	5	8	11	14
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	3	5	8	11	14
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	3	5	8	11	14
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	3	5	8	11	13
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	3	5	8	11	13
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	3	5	8	11	13
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	3	5	8	11	13
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	3	5	8	10	13
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	3	5	8	10	13
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	3	5	8	10	13
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	3	5	8	10	13
30	0.500	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	3	5	8	10	13
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	2	5	7	10	12
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	2	5	7	10	12
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	2	5	7	10	12
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	2	5	7	10	12
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	2	5	7	9	12
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	2	5	7	9	12
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	2	5	7	9	12
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	2	5	7	9	11
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	2	4	7	9	11
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	2	4	7	9	11
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	2	4	7	9	11
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	2	4	6	9	11
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0,6921	0.6934	2	4	6	8	11
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	2	4	6	8	10
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	2	4	6	8	10
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	2	4	6	8	10
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	2	4	6	8	10
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	2	4	6	8	10
49	0.7547	0.7558	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	2	4	6	8	9
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	2	4	6	7	9
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	2	4	5	7	9
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	2	4	5	7	9
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	2	3	5	7	9
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	2	3	5	7	8
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	2	3	5	7	8
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	2	3	5	6	8
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	2	3	5	6	8
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	2	3	5	6	8
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	1	3	4	6	7

Natural Sines

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	1	3	4	6	7
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	1	3	4	6	7
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	1	3	4	5	7
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	1	3	4	5	6
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	1	3	4	5	6
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	1	2	4	5	6
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	1	2	3	5	6
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	1	2	3	4	6
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	1	2	3	4	5
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	1	2	3	4	5
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	1	2	3	4	5
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	1	2	3	4	5
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	1	2	3	3	4
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	1	2	2	3	4
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	1	2	2	3	4
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	1	1	2	3	4
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	1	1	2	3	3
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	1	1	2	2	3
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	1	1	2	2	3
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	1	1	2	2	3
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0	1	1	2	2
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0	1	1	2	2
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0	1	1	1	2
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0	1	1	1	2
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0	1	1	1	1
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0	0	1	1	1
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0	0	1	1	1
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0	0	0	1	1
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0	0	0	0	0
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0
90	1.0000														

Fig 1



SINE CURVE

Quadrant	Angle	sinA =	Examples
First	0 to 90°	sin A	sin 34°38' = 0.5683
Second	90° to 180°	sin(180° – A)	sin 145°22' = sin(180° – 145° 22') = sin 34°38' = 0.5683
Third	180° to 270°	–sin(A – 180°)	sin 214°38' = –sin(214°38' – 180°) = –sin34°38' = –0.5683
Fourth	270° to 360°	–sin(360° – A)	sin 325°22' = –sin(360° – 325°22') = – sin 34°38' = –0.5683

Natural Cosines

Numbers in different columns to be subtracted**, not added**

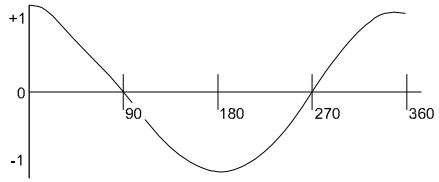
°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0	0	0	0	0
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0	0	0	0	0
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0	0	0	1	1
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977	0	0	1	1	1
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963	0	0	1	1	1
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947	0	1	1	1	1
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928	0	1	1	1	2
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905	0	1	1	1	2
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880	0	1	1	2	2
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851	0	1	1	2	2
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820	1	1	2	2	3
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785	1	1	2	2	3
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748	1	1	2	2	3
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707	1	1	2	3	3
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664	1	1	2	3	4
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617	1	2	2	3	4
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568	1	2	2	3	4
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516	1	2	3	3	4
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461	1	2	3	4	5
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403	1	2	3	4	5
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342	1	2	3	4	5
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278	1	2	3	4	5
22	0.9272	0.9625	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212	1	2	3	4	6
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143	1	2	3	5	6
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070	1	2	4	5	6
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996	1	3	4	5	6
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918	1	3	4	5	6
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838	1	3	4	5	7
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755	1	3	4	6	7
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669	1	3	4	6	7
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581	1	3	4	6	7
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490	2	3	5	6	8
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396	2	3	5	6	8
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300	2	3	5	6	8
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202	2	3	5	7	8
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100	2	3	5	7	8
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997	2	3	5	7	9
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891	2	4	5	7	9
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782	2	4	5	7	9
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672	2	4	6	7	9
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559	2	4	6	8	9
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443	2	4	6	8	10
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325	2	4	6	8	10
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206	2	4	6	8	10
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083	2	4	6	8	10
45	0.7071	0.7059	0.7046	0,7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959	2	4	6	8	10
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833	2	4	6	8	11
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704	2	4	6	9	11
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574	2	4	7	9	11
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441	2	4	7	9	11
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307	2	4	7	9	11
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6255	0.6211	0.6198	0.6184	0.6170	2	5	7	9	11
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6404	0.6060	0.6046	0.6032	2	5	7	9	12
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892	2	5	7	9	12
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750	2	5	7	9	12
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606	2	5	7	10	12
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461	2	5	7	10	12
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314	2	5	7	10	12
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165	2	5	7	10	12
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015	3	5	8	10	13

Natural Cosines

Numbers in different columns to be subtracted**, not added**

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863	3	5	8	10	13
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710	3	5	8	10	13
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555	3	5	8	10	13
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399	3	5	8	10	13
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242	3	5	8	11	13
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083	3	5	8	11	13
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923	3	5	8	11	13
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762	3	5	8	11	13
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600	3	5	8	11	14
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437	3	5	8	11	14
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272	3	5	8	11	14
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107	3	6	8	11	14
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940	3	6	8	11	14
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773	3	6	8	11	14
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605	3	6	8	11	14
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436	3	6	8	11	14
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267	3	6	8	11	14
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096	3	6	9	11	14
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925	3	6	9	11	14
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754	3	6	9	11	14
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582	3	6	9	11	14
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409	3	6	9	11	14
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236	3	6	9	12	14
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063	3	6	9	12	14
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889	3	6	9	12	14
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715	3	6	9	12	14
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541	3	6	9	12	15
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366	3	6	9	12	15
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192	3	6	9	12	15
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017	3	6	9	12	15
90	0.0000														

Fig 2



COSINE CURVE

Quadrant	Angle	cos A =	Examples
First	0 to 90°	cos A	cos 33°26' = 0.8345
Second	90° to 180°	–cos(180° – A)	sin 146°34' = –cos(180° – 146o 34') = –cos 33o26' = –0.8345
Third	180° to 270°	cos(A – 180°)	cos 213°26' = –cos(213°26' – 180°) = –cos 33°26' = –0.8345
Fourth	270° to 360°	cos(360° – A)	cos 326°34' = cos(360° – 326°34') = cos 33°26' = 0.8345

Natural Tangents

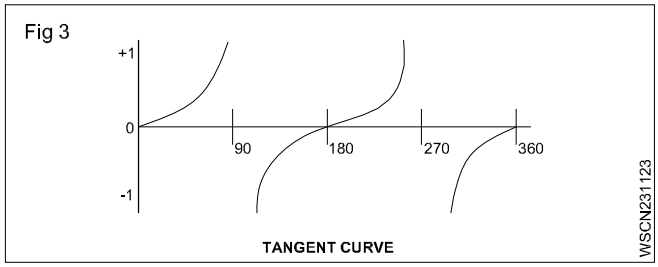
°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	3	6	9	12	15
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	3	6	9	12	15
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	3	6	9	12	15
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	3	6	9	12	15
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	3	6	9	12	15
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	3	6	9	12	15
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	3	6	9	12	15
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	3	6	9	12	15
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	3	6	9	12	15
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	3	6	9	12	15
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	3	6	9	12	15
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	3	6	9	12	15
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	3	6	9	12	16
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	3	6	9	13	16
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	3	6	9	13	16
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	3	6	10	13	16
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	3	6	10	13	16
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	3	7	10	13	16
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	3	7	10	13	17
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	3	7	10	13	17
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	3	7	10	14	17
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	3	7	10	14	17
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	4	7	11	14	18
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	4	7	11	14	18
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	4	7	11	15	18
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	4	7	11	15	18
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	4	8	11	15	19
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	4	8	12	15	19
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	4	8	12	16	20
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	4	8	12	16	20
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	4	8	12	16	20
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	4	8	13	17	21
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	4	9	13	17	21
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	4	9	13	17	22
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	5	9	14	18	23
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	5	9	14	18	23
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	5	9	14	19	24
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	5	10	15	20	24
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	5	10	15	20	25
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	5	10	16	21	26
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	5	11	16	21	27
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	6	11	17	22	28
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	6	11	17	23	28
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	6	12	18	24	30
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	6	12	18	25	31
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	6	13	19	25	32
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	7	13	20	27	33
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	7	14	21	28	34
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	7	14	22	29	36
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	8	15	23	30	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	8	16	24	31	39
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	8	16	25	33	41
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	9	17	26	34	43
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	9	18	27	36	45
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	10	19	29	38	48
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	10	20	30	40	50
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	11	21	32	43	53
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	11	23	34	45	56

Natural Tangents

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	12	24	36	48	60
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	13	26	38	51	64
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	14	27	41	55	68
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	16	31	47	63	78
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	17	34	51	68	85
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	18	37	55	73	92
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	20	40	60	79	99
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	22	43	65	87	108
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	24	47	71	95	119
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	26	52	78	104	131
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506	32	64	96	129	161
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	36	72	108	144	180
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	41	81	122	163	204
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	46	93	139	186	232
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	53	107	160	213	267
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646					
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970					
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140					
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264					
82	7.1154	7.2066	7.3002	6.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285					
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					
90	×														

DIFFERENCES UNTRUSTWORTHY HERE

Fig 3



TANGENT CURVE

Quadrant	Angle	tan A =	Examples
First	0 to 90o	tan A	tan 56o17' = 1.4986
Second	90o to 180o	–tan(180o – A)	tan 123o43' = –tan(180o – 123o 43') = –tan 56o17' = –1.4986
Third	180o to 270o	tan(A – 180o)	tan 236o17' = tan(236o17' – 180o) = tan 56o17' = 1.4986
Fourth	270o to 360o	–tan(360o – A)	tan 303o43' = –tan(360o – 303o43') = – tan 56o17' = –1.4986

Heights and distances

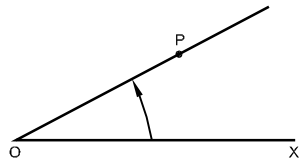
One of the practical applications of Trigonometry is to find distances and heights of distant and inaccessible objects. Two angles are often used in the practical applications of Trigonometry and they are defined as follows:

The angle between a horizontal plane through an observer's eye and line joining the eye to an object is called

i **The angle of elevation** when the object is higher than the eye. (Fig 1)

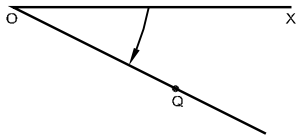
ii **The angle of depression** when the object is lower than the eye. (Fig 2)

Fig 1



WSNA371111

Fig 2



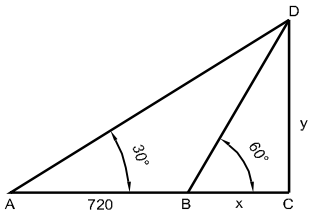
WSNA371112

If OX be a horizontal line through 'O' the observer's eye and 'P' any point above OX, then XOP is the angle of elevation of P at O. If 'Q' below OX, then XOQ is called the angle of depression of Q at O.

Example

1 At a certain point on the ground, it is found that the angle of elevation of the top of a tower is 30°. On walking 720 m. towards the foot of tower, the angle of elevation is found to be 60°. Find the height of the tower.

Fig 3



WSNA371113

Let CD be the tower and A and B be the points from which the tower is observed and let BC be x and CD be y.

In triangle ADC,

$$\tan 30^\circ = \frac{CD}{AC} = \frac{y}{x+720}$$

$$y = (x + 720)\tan 30^0$$

$$\text{Angle BDC, } \tan 60^\circ = \frac{CD}{BC} = \frac{y}{x}$$

$$\therefore y = x\tan 60^\circ$$

$$\text{Hence, } (x + 720)\tan 30^\circ = x\tan 60^\circ$$

$$(x+720)\frac{1}{\sqrt{3}} = x\sqrt{3}$$

$$\text{i.e. } \frac{x+720}{\sqrt{3}} = \sqrt{3}x$$

$$\therefore x + 720 = 3x$$

$$720 = 3x - x$$

$$2x = 720$$

$$x = \frac{720}{2} = 360$$

$$\text{Hence } y = 360\tan 60^\circ$$

$$= 360\sqrt{3}$$

$$\tan 60^0 = \frac{y}{x}$$

$$\tan 60^0 = \frac{y}{360}$$

$$y = \tan 60^0 \times 360$$

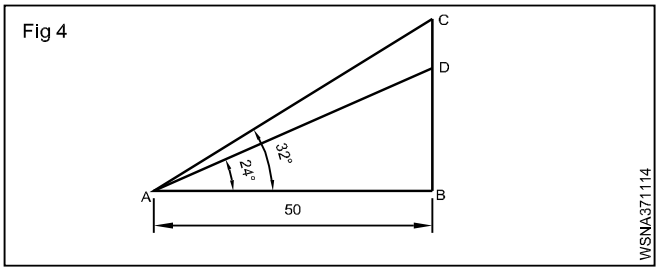
Hence the height of the tower is $360\sqrt{3}$

$$= 360 \times 1.732$$

Height of the tower = 623.5 metres

2 A flag pole stands on the top of a building when viewed from a distance of 50 m. (measured horizontally) the angle of elevation of the top and bottom of the flag staff are 24 and 32° respectively. Find the height of the flag pole.

Fig 4



WSNA371114

Let CD be the flag pole, A the point of observation and B a point on the same level as A and directly underneath the flag pole.

In the triangle DAB

$$\tan 24^\circ = \frac{BD}{AB} = \frac{BD}{50}$$

$$\therefore BD = 50.\tan 24^\circ$$

$$= 50 \times 0.4452$$

$$= 22.26$$

In the triangle CAB,

$$\tan 32^{o} = \frac{CB}{AB} = \frac{BC}{50}$$

$$\therefore BC = 50 \,.\, \tan 32^{o}$$

$$= 50 \times 0.6249$$

$$= 31.25$$

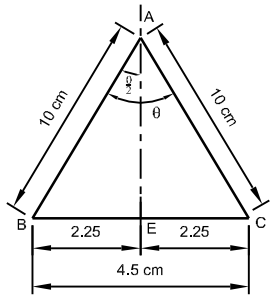
$$\text{Hence } DC = 31.25 - 22.26$$

$$= 8.99$$

$$= 9 \text{ m nearly.}$$

3 A divider having legs of equal length of 10 cm is opened so that its points are 4.5 cm apart. Using trigonometrical tables. Find the angle between the legs.

Fig 5



WSNA371115

Distance between the two legs of divider = BC = 4.5 cm

AC = AB ; Length of leg = 10 cm

The perpendicular line drawn from the centre of BC, (point E) to point A makes two right angled triangle ABE and AEC. If the angle between two legs of divider is θ

$$\angle BAE = \frac{\theta}{2}$$

$$= Sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{Sin}\frac{\theta}{2} = \frac{BE}{AB}$$

$$= \frac{2.25}{10} = 0.225$$

Find the **θ** value of 0.225 from sin table

$0.225 = 13^{0}$

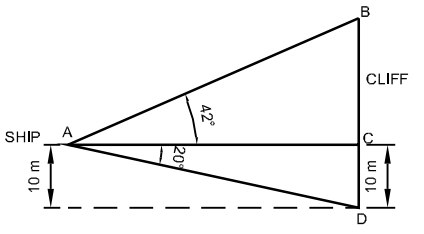
$\frac{\theta}{2} = 13^{0}$

$\theta = 13^{0} \times 2 = 26^{o}$

Angle between two leg of divider is 26°

4 A man on the deck of a ship is 10 m above water level. He observes that angle of elevation of a cliff is 42° and angle of depression of its base is 20°. Calculate (i) the distance of the cliff from the ship. (ii) the height of the cliff.

Fig 6



WSNA371116

Find the distance between ship and cliff

From right angled triangle CAD

$$tan\,\theta = \frac{\text{Opposite side}}{\text{Adjacent Side}} = \frac{CD}{AC}$$

$$\tan 20^{o} = \frac{10}{AC}$$

$$0.3640 = \frac{10}{AC}$$

$$AC = \frac{10}{0.3640} = 27.473 \text{ m}$$

Height of cliff BD = DC + CB

Find CB in right angled triangle BAC

$$tan\,\theta = \frac{\text{Opposite side}}{\text{Adjacent Side}} = \frac{BC}{AC}$$

$$\tan 42^{o} = \frac{BC}{27.473}$$

$$0.9004 = \frac{BC}{27.473}$$

$BC = 0.9004 \times 27.473$

$= 24.737 \text{ m}$

$\text{Height of cliff BD} = DC + CB$

$= 10 + 24.737$

$= 34.737 \text{ m}$

Distance between ship and cliff = 27.473 m

Height of cliff = 34.737 m

5 The angle of depression from the top of a hill to the bottom of a tower is 60° and the angle of depression from the top of the tower to the bottom of hill is 30°. If the height of tower is 50m, then find the height of the hill.

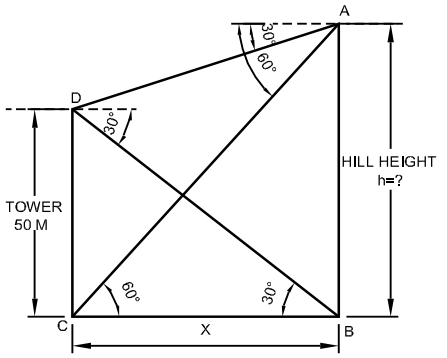
Note :

1 If the angle of depression at A is 60°, then the angle of elevation at C is 60°.

2 If the angle of depression at D is 30°, then the angle of elevation at B is 30°.

Let, the height of hill is 'h' and distance between bae of hill and base of tower is x.

Fig 7



From right angled triangle ACB

$$tan\ \theta = \frac{\text{Opposite side}}{\text{Adjacent Side}}$$

$$\tan 60^{o} = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}x \quad \text{..... equation 1}$$

From right angled triangle DBC

$$\tan 30^{\circ} = \frac{50}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{50}{x}$$

$$x = 50x\frac{\sqrt{3}}{1}$$

$$x = 50\sqrt{3} \quad \text{.....equation 2}$$

Substitute the value of = $x = 50\sqrt{3}$ in equation 1.

$$h = \sqrt{3}\ x\ x$$

$$= \sqrt{3} \times 50\sqrt{3}$$

$$= 3 \times 50 = 150 \text{ m}$$

Height of hill = 150 m

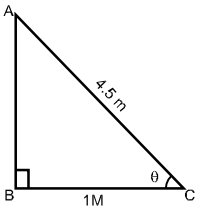
6 The foot of a 4.5 m long ladder is placed at 1 m away from the wall. Find the angle which the ladder makes with the ground

In Right angled Δ

$$\text{Cos C} = \frac{BC}{AC}$$

$$\text{Cos } \theta = \frac{1 \text{ m}}{4.5 \text{ m}}$$

Fig 8



$$\cos \theta = 0.2222$$

$$0.2233 = \cos 77^{o}\ 6'$$

$$(-)\quad 0.0011 = \qquad 4'\ (+)$$

$$0.2222 = \cos 77^{o}\ 10'$$

$$\theta = 77^{o}10'$$

Ladder makes the angle = 77° 10'

7 A Line man who is working on a road places his ladder which is 12 m in length at a point on the road such that it makes an angle of 60^{0} with the ground, when it is placed against a lamp placed against another lamp post directly on the opposite side of the road it makes an angle of 30^{0}. Find the distance between the 2 lamp posts.

AC and CD are Ladder

AB and DE are lamp post

In Δ ABC

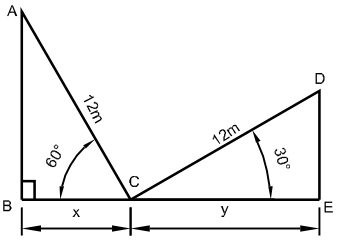
$$\text{Cos } 60^{o} = \frac{BC}{AC} = \frac{x}{12 \text{ m}}$$

$$\therefore x = \cos 60^{o} \times 12 \text{ m}$$

$$= 0.5000 \times 12 \text{ m}$$

$$= 6 \text{ m}$$

Fig 9



In Δ CDE

$$\text{Cos } 30^{o} = \frac{CE}{CD} = \frac{y}{12 \text{ m}}$$

$$\therefore y = \cos 30^{o} \times 12 \text{ m}$$

$$= 0.8660 \times 12 \text{ m}$$

$$= 10.392 \text{ m}$$

Distance between two lamp posts

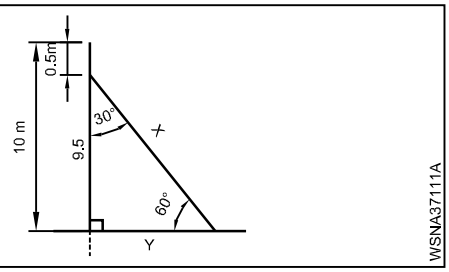
$= x + y$

$= 6 + 10.392$ m

Distance between two lamp posts = 16.392 m

8 A pole stands 10 metre above the ground and stay wire is fixed to the pole at 0.5 metre from the top. If the stay wire is to make an angle of 60^0 with the horizontal, find the distance of stay rod from the base of the pole. Also find the length of the stay wire.

Fig 10



stay wire fixed 0.5m below from the top of the pole

$$\text{Sin}\,\theta = \frac{\text{OPP}}{\text{HYP}}$$

$$\text{Sin } 60^o = \frac{9.5}{x}$$

x x sin 60^o = 9.5 m

$$x = \frac{9.5}{\text{Sin } 60^\circ}$$

$$= \frac{9.5}{0.8660} = 10.9699$$

Stay wire (x) = 10.97 m

$$\tan\,\theta = \frac{\text{Opp}}{\text{Adj}}$$

$$\tan 60^o = \frac{9.5}{y}$$

$$y \tan 60^o = 9.5$$

$$y = \frac{9.5}{\tan 60^\circ}$$

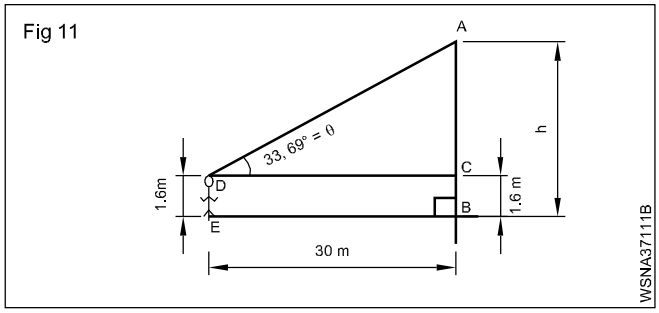
$$= \frac{9.5}{1.7321} = 5.48 \text{ m}$$

Length of the stay wire = 10.97 metre

distance of the stay wire from the base of the pole = 5.48 metre

9 An electrician standing 30metres away from the base of a transmission line tower looks at the top of the tower. His line of sight makes an angle of 33.69^0 with the horizontal. If the height of his eyes above the ground level is 1.6metres. Find the height of the tower?

Fig 11



Given:

Distance between the Electrician and the tower = EB = DC = 30m

Height of his eyes above the ground level = CB = DE = 1.6m

Height of the tower AB (h) = ?

$33.69^0 = 33^0 + (0.69 \times 60')$

$= 33^0 + 41'$

$33.69^0 = 33^0 41'$

In Right angled triangle ADC

$$\text{Tan}\,\theta = \frac{AC}{DC}$$

AC = tan θ x DC

= tan $33^0 41'$ x 30

= 0.6665 x 30 m

= 19.995 m

Height of the tower AB (h) = AC + CB

= 19.995 + 1.6

Height of the tower (h) = 21.595 m

1 A person walking along straight observed that at consecutive mile stones the angle of elevation of a hill in front of him 30° and 75°. Find the height of the hill.

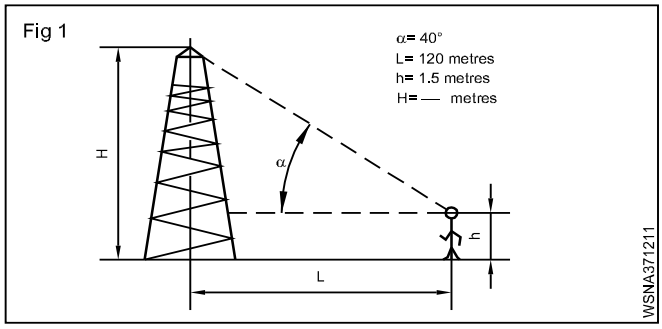
2 A shadow of an electric pole is reduced by 4 metres when the sun changes its angle of elevation from 30° to 45°. If the pole is burried in the ground by 2 metres, find the total length of the pole?

3 The angle of elevation of the top on an unfinished tower at a point distance 120 metre from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same may be 60°.

4 Two objects on horizontal plane in the same line of the foot of the cliff form with the top of cliff angles of elevation of 30° and 45°. If the height of the cliff is 100 m, calculate the distance between the two objects.

5 From a point 'A' on the ground at unknown distance from the bae of the radio tower, the angle of elevation of the top of mast is 65°. Proceeding in the same straight line to the point 'B' 50 m from 'A', the angle of elevation is reduced to 50°. Find the height of the mast and distance of 'A' from the mast.

6



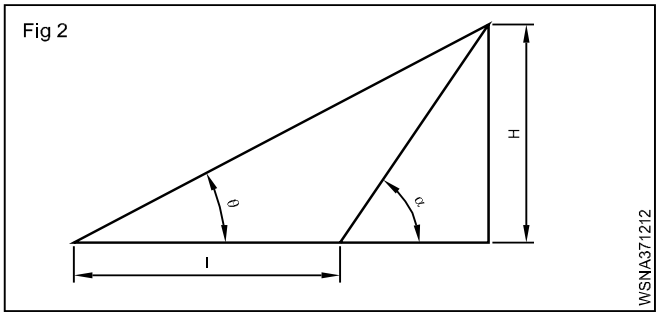
α = 40°

L = 120 metres

h = 1.5 metres

H = _____ metre

7



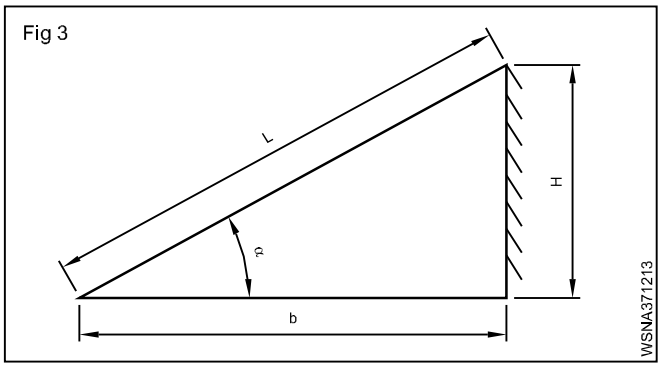
θ = 25°

α = 50°

l = 100 metres

H = _____ metre

8



α = 75°

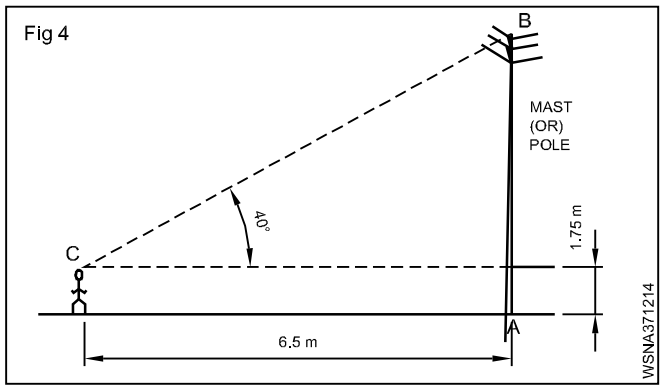
b = 3 metres

H = _____ metres

L = _____ metres

9 The height of the pole,

AB = _____ m.



10 Find the values of the given angles.

1 Sin 65°

2 Sin 42° 23'

3 Sin 66° 35' 32"

4 Cos 47° 39'

5 Tan 28° 45'

11 Find corresponding angles for given values.

1 Sin θ = 0.3062

2 Sin θ = 0.04802

3 Cos θ = 0.6446

4 Tan θ = 0.3411

6 Tan θ = 2.3868

12 The slant height of a cone is 12.25 cm and the vertex angle is 110^0. Calculate its base.

13 A ladder 2.5 m long makes an angle of 60^0 with the ground. Find the height of the wall where the ladder touches the wall.