Any object comprises of a large number of particles. Each particle is pulled towards the earth due to the force of gravity. Thus, the forces on the particles are equal, parallel and act in the same direction. These forces will have a resultant which acts through a particular point 'G'. This fixed point 'G' is called the centre of gravity.

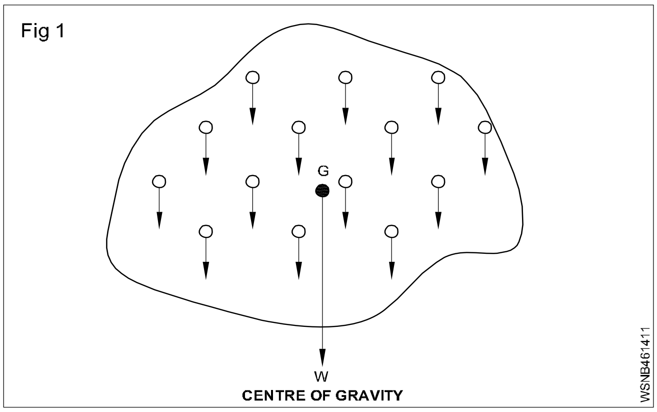


Fig 1

Concept of Centre of gravity

In physics, an imaginary point in a body of matter where, for convenience in certain calculations, the total weight of the body may be thought to be concentrated. The concept is sometimes useful in designing static structures (e.g., buildings and bridges) or in predicting the behaviour of a moving body when it is acted on by gravity.

In a uniform gravitational field the centre of gravity is identical to the centre of mass, a term preferred by physicists.

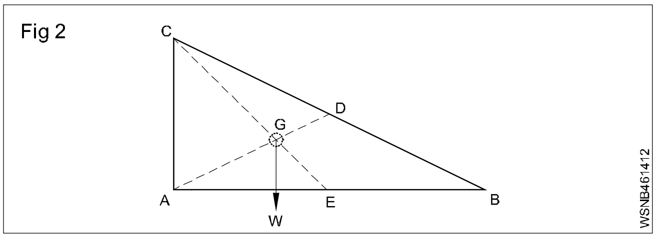


Fig 2

Gravitation

The mutual attractive force of bodies due to which they attract each other is called gravitation.

1 Gravity

The attractive force of the earth due to which it attracts all bodies towards its centre is called gravity.

The value of gravity varies from place to place on the ground surface. Its general value is 9.81 m/s^2.

Centroid

Different geometrical shapes such as the circle, triangle and rectangle are plane figures having only 2-dimensions. They are also known as laminas. They have only area, but no mass. The centre of gravity of these plane figures is called as the Centroid. It is also known as the geometrical centre. The method of finding out the centroid of a plane figure is the same as that of finding out the centre of gravity of a body. If the lamina is assumed to have uniform mass per unit area, then the centroid is also the centre of gravity in a uniform gravitational field.

Methods to calculate centre of gravity

1 By geometrical consideration.

2 By moments.

Principle : The total moment of a weight about any axis = The sum of the moments of the various parts about the same axis.

3 By graphical method.

The first two methods are generally used to find out the centre of gravity or centroid, as the third method can become tedious.

Centre of gravity by geometrical consideration

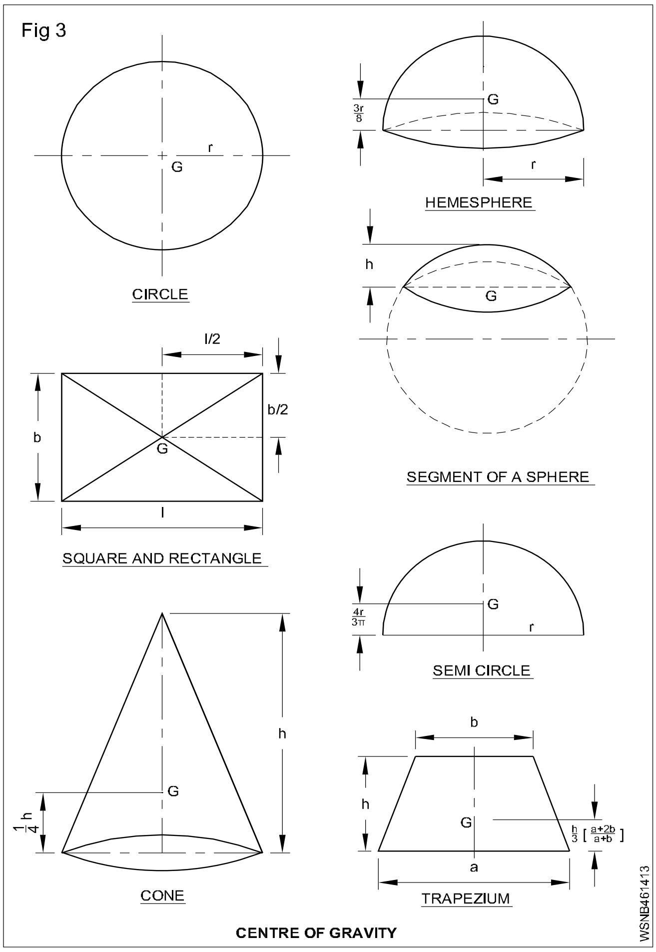


Fig 3

1 The centre of gravity of a circle is its centre.

2 The centre of gravity of a square, rectangle or a parallelogram is at the points where its diagonals meet each other. It is also the middle point of the length as well as the width.

3 The centre of gravity of a triangle is at the point where the medians of the triangle meet.

4 The centre of gravity of a right circular Cone is at a distance of from its base.

5 The centre of gravity of a hemisphere is at a distance of from its base.

6 The centre of gravity of a segment of a sphere of radius h is at a perpendicular distance of from the centre of the sphere.

7 The centre of gravity of a semicircle is at a perpendicular distance of from its centre.

8 The centre of gravity of a trapezium with parallel side 'a' and 'b' is at a distance of measured from the base 'b'.

9 The centre of gravity of a cube of side L is at a distance of from every face.

10 The centre of gravity of a Sphere of diameter 'd' is at a distance of from every point.

Centre of gravity; An experiment

- Number of 2 pencil
- A fine edge like a ruler or a credit card
- A permanent marker
- A ruler

Step 1

Attempt to balance the pencil on the edge you have selected

Balancing the pencil may take some trial and error. The point at which the pencil balances may not be where you first thought. If it begins to tip in one direction, move the pencil back slowly in the opposite direction until it will stay there on its own.

Step 2

Once the pencil is balanced, mark the location of the balancing point with a permanent marker.

Step 3

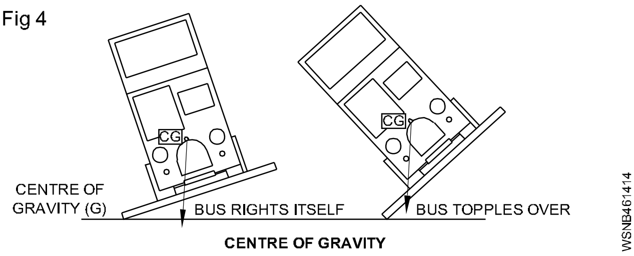
Measure the distance between the ends of the pencil and the balancing point you have marked. Are the two lengths equal? On my pencil, the length from the eraser to the balancing point was actually 1.25 inches less than the length from the pencil tip to the balancing point. Why would this be the case?

In our experiment, the balancing point was another word for the centre of gravity of this pencil. In other words, if we cut the pencil in two at the mark we made in the experiment, the two parts would be equal in weight. However, they are not equal in length. As you may have already figured out, the metal piece that houses the eraser contributes more to the weight of the pencil, so the CG is closer to that side of the pencil.

Keeping up with that centre

The centre of gravity is an important concept in determining the stability of a structure. It’s the reason why a good homeowner will keep the top branches of his trees trimmed. It’s also the reason why a pick-up truck might not be the best vehicle choice for a first time driver. Stability is maximized in objects with a lower centre of gravity and a wide base. The taller and more top-heavy an object, the more likely it is to tip over when it is tilted by a force. This figure demonstrates a bus driving on two different grades; the second one is steep enough to cause the centre of gravity to fall outside of the base of the vehicle, which will cause it to topple over.

Fig 4



Equilibrium

A body is said to be in equilibrium if the resultant of all the forces acting on a body is zero and if there is no turning moment.

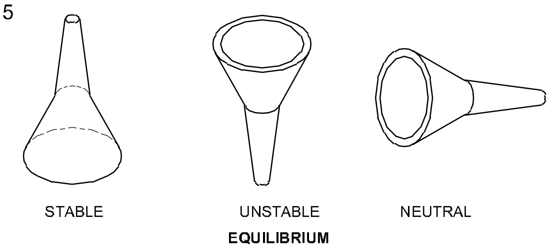
There are three states of equilibrium (Fig 5)

1 Stable equilibrium

2 Unstable equilibrium

3 Neutral equilibrium

Fig 5



1 Stable equilibrium

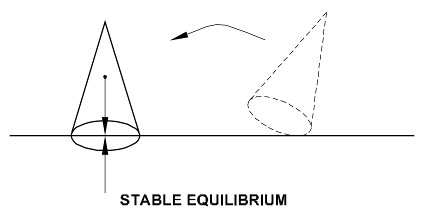
A body is said to be in a stable equilibrium if it returns to its original position when slightly displaced. (The C.G. is as low as possible).

E.g : 1 A cone resting on its base

2 A ball on a concave surface

3 Funnel resting on its base. (Fig 6)

Fig 6



2 Unstable equilibrium

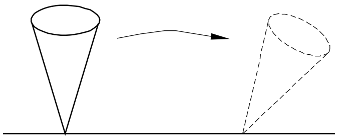
A body is said to be in an unstable equilibrium if it does not return to its original position when slightly displaced. Its centre of gravity falls taking it away from its original position. (CG is at high points)

E.g: 1 A cone resting on its tip

2 A ball on convex surface

3 Funnel standing on its tube end. (Fig 7)

Fig 7



UNSTABLE EQUILIBRIUM

WSNB461417

3 Neutral equilibrium

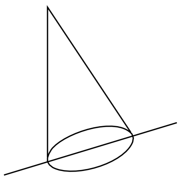
A body is said to be in a neutral equilibrium if on being slightly displaced, it takes a new position similar to its original one. The centre of gravity remains undisturbed. (CG is neither raised or lowered)

Eg: 1 A cone resting on its side

2 A ball on flat surface

3 Funnel resting on its side (Fig 8)

Fig 8



UNSTABLE EQUILIBRIUM

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Model 1

Conditions for stable equilibrium

- The CG should be as low as possible.
- It should have a broad base.
- The vertical line passing through the CG should fall within the base.

Conditions of equilibrium

A body is said to be in a state of equilibrium under the action of forces when there is no motion of rotation or translation of the body. There are three conditions of equilibrium of a body which are given below:

i Algebraic sum of the horizontal components of all the forces acting on the body must be zero.

$\Sigma H = 0$

ii Algebraic sum of the vertical components of all the forces acting on the body must be zero.

$\Sigma V = 0$

iii Algebraic sum of the moments of all the forces acting on the body must be zero.

$\Sigma M = 0$

Torque or twisting moment of a couple is given by the product of force applied and the arm of the couple (i.e. Radius). In fact, moment means the product of "force applied" and the "perpendicular distance of the point and the line of the force".

Some example of equilibrium in daily life

1 The lower decks of the ships are loaded with heavy cargoes. This makes the centre of gravity of the whole ship lower and its equilibrium becomes more stable.

2 A man carrying a bucket full of water in one hand extends his opposite arm and bends his body towards it.

3 While carrying load on back the man bends forward so that his and the load's centre of gravity falls on his feet, if he walks erect, he will fall backward.

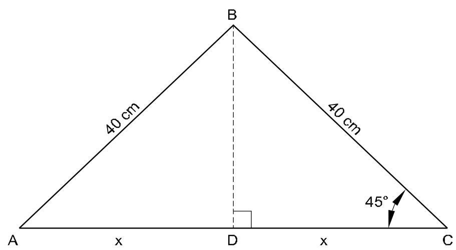
4 While climbing a mountain, a man bends forward and bends backward while descending so that the centre of gravity of his load falls on his feet.

5 In a double-decker, more passengers are accommodated in the lower deck and less on the upper so that the centre of gravity of the bus and the passengers is kept low to eliminate any chance of turning.

Example

1 Find the centroid of the isosceles triangular plate as shown in the figure.

Fig 1



CENTRE OF GRAVITY

WSNB461421

Since $\angle BCD=45^\circ$ then BD=DC=x

As per Pythagoras theorem

$$BD^2 + DC^2 = CB^2$$

$$x^2 + x^2 = 40^2$$

$$2x^2 = 1600$$

$$x^2 = \frac{1600}{2} = 800$$

$$x = \sqrt{800} = 28.28\text{ cm}$$

$$\text{Centroid from BD} = \frac{x}{3} = \frac{28.28}{3} = 9.43\text{ cm}$$

2 A rectangular lamina has 10 cm and 8 cm. Find the centroid. Centroid of rectangular = Diagonals intersecting point.

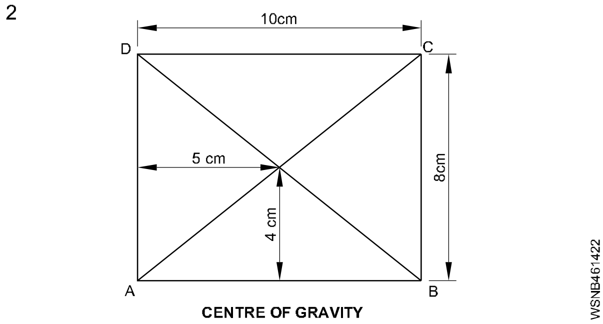
Centroid of rectangular = Diagonals intersecting point

$$\text{Centre of AB} = \frac{10}{2} = 5$$

$$\text{Centre of AB} = \frac{8}{2} = 4$$

Centroid lying 4 cm from AB and 5 cm from AD

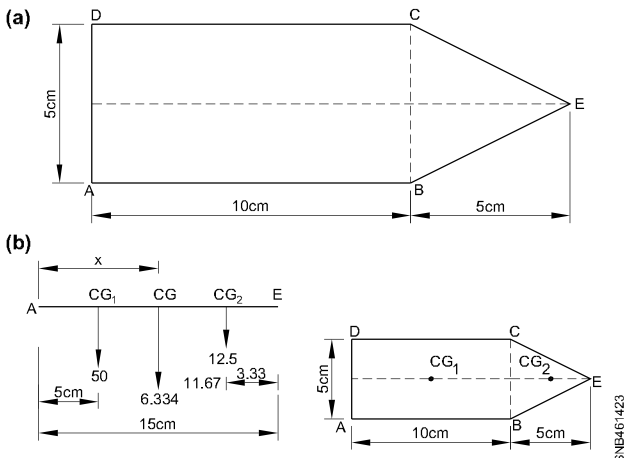
Fig 2



CENTRE OF GRAVITY

3 A thin lamina is shown in the figure below. Find the centre of gravity.

Fig 3



CENTRE OF GRAVITY

Centroid of rectangle

Area of rectangle= 5 x 10 = 50cm^2

Area of triangle $= \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 5 = 12.5\ cm^2$

Total area $= 50 + 12.5 = 62.5\ cm^2$

The centre of gravity for rectangle is the point of intersection of diagonal = 5 cm distance from AD (CG_1)

Centre of gravity for triangle is $\frac{1}{3}$ distance from its height.

$= 5 \times \frac{1}{3} = \frac{5}{3} = 1.67$ cm

(CG_2) Centroid of plate is lying in between CG_1 and CG_2. From the figure torque is about AD.

$62.5x = 50 \times 5 + 12.5 \times 11.67$

$= 250 + 145.875$

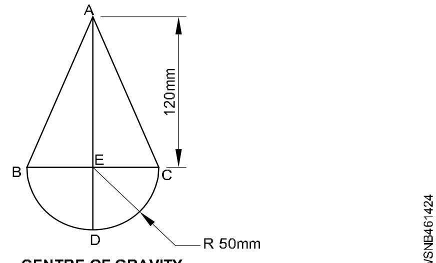
$62.5x = 395.875$

$x = \frac{395.875}{62.5} = 6.334$ cm

Centre of gravity is 6.334 cm from AD, on the centre axis.

4 A thin lamina consists of an isosceles triangle of height 120mm and base 100mm placed on a semicircle of diameter 100mm. find the location of its centre of gravity.

Fig 4



CENTRE OF GRAVITY

Area of right angled triangle (a_1) $= \frac{1}{2}bh$

$= \frac{1}{2} \times 10 \times 12$

$= 60\ cm^2$

Centroid of right angled triangle $= \frac{1}{3}h$ from base

$= \frac{1}{3} \times 12$

Centroid from E = 4 cm

Centroid from A (h_1) = 12 - 4 = 8 cm

Area of half circle (a_2) $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times 3.14 \times 5 \times 5$

$= 39.25\ cm^2$

Centroid of semi circle = (Vertical distance from centre of diagonal)

$= \frac{4r}{3\pi}$

Centroid distance from E to D $= \frac{4 \times 5}{3 \times 3.14}$

= 2.123 cm

$$(h_2) = \begin{pmatrix}\text{Height of}\\ \text{triangle}\end{pmatrix} + \begin{pmatrix}\text{Centroid of}\\ \text{half circle}\end{pmatrix}$$

= 12 + 2.123

= 14.123 cm

To find centroid of lamina

$$y = \frac{a_1 h_1 + a_2 h_2}{a_1 + a_2}$$

$$= \frac{60 \times 8 + 39.25 \times 14.123}{60 + 39.25}$$

$$= \frac{480 + 554.328}{99.25}$$

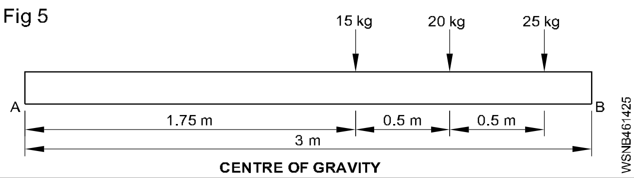
$$= \frac{1034.328}{99.25}$$

= 10.421 cm

Centroid is lying at 10.421 cm from point A

5 A uniform rod weighing 50kg and 3m long carries loads as shown below. Find out the distance of the CG of the system from the left hand end.

Fig 5



CENTRE OF GRAVITY

Distance of CG from A = x

Total weight = 50 + 15 + 20 + 25 = 110 kg

110 x x = (50 x 1.5) + (15 x 1.75) + (20 x 2.25) + (25 x 2.75)

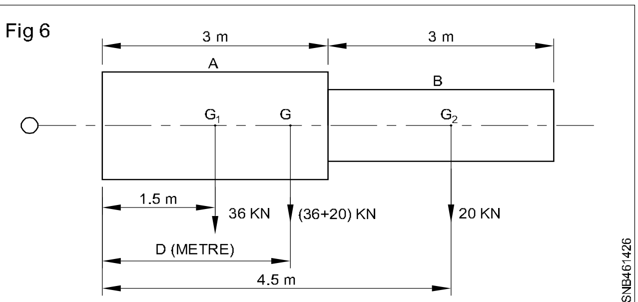
= 75 + 26.25 + 45 + 68.75 = 215

Therefore $x = \frac{215}{110} = 1.96$ m

Distance of CG of the system from A = 1.95 m

6 A long shaft is composed of two section A and B each 3 meter long and weight 36KN and 20KN respectively. Find out the position of centre of gravity of the shaft.

Fig 6



CENTRE OF GRAVITY

Solution

Let G_1 be the c.g. point of section A

Let G_2 be the common c.g. of the shaft and its distance is D from left hand end.

Now, take moments about 'O'

A Moment of section A about O = 36 KN x 1.5 m

Moment of section B about O = 20 KN x 4.5 m

Adding both we get as below

Total moment about O=(36 KN x 1.5 m + (20KN x 4.5m)

= 54 KNm + 90KNm

= 144 KNm

B This moment is equal to moment of section A and section B about 'O' (distance of action being D meter)

That is =(36KN+20KN) x D(meter)=56 DKNm

Again equating A and B

144 KNm = 56 DKNm

$$\frac{144 \text{ KNm}}{56 \text{ KNm}} = D$$

$$\frac{144}{56} = D$$

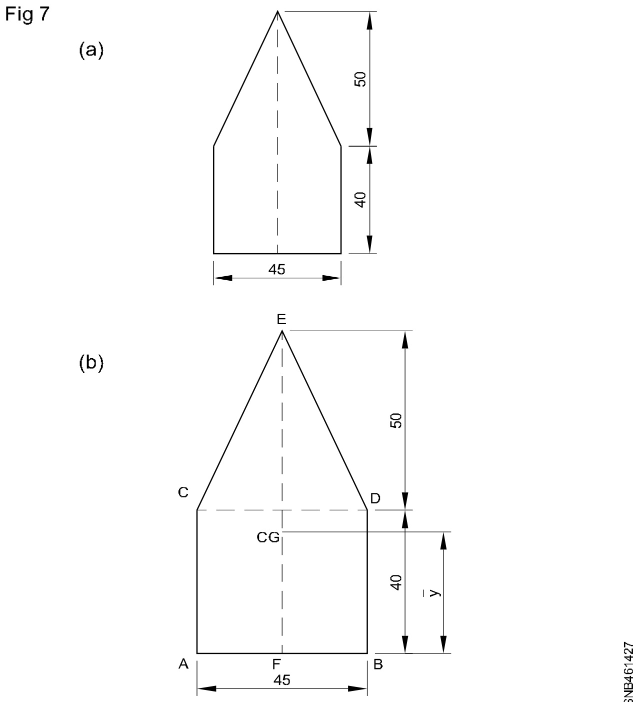
Therefore $D = \frac{18}{7}$

= 2.57 meters

The distance of CG of the shaft from left hand is 2.57 meters.

7 A thin lamina is shown in the figure. Find centre of gravity.

Fig 7



CENTRE OF GRAVITY

As the body is symmetrical about y-axis centre of gravity lies on this axis.

Let AB is the axis of reference

Let y = The distance between centre of gravity and point F, the point of reference as shown in the figure.

Let a_1= Area of rectangle CDBA = 45 x 40 = 1800 mm^2

h_1 = Distance between centre of gravity of rectangle

of point F = $\frac{40}{2}$ = 20 mm

Let a_2 = Area of triangle ECD=1/2 x base x height

=1/2 x 45 x 50 = 1125 square mm

h_2 = distance between centre of gravity of triangle of point F.

=1/3rd height of triangle +width of rectangle

$$= \frac{1}{3}(50) + 40 = \frac{50}{3} + 40 = \frac{170}{3} \text{ mm}$$

Applying formula

$$y = \frac{a_1 h_1 + a_2 h_2}{a_1 + a_2}$$

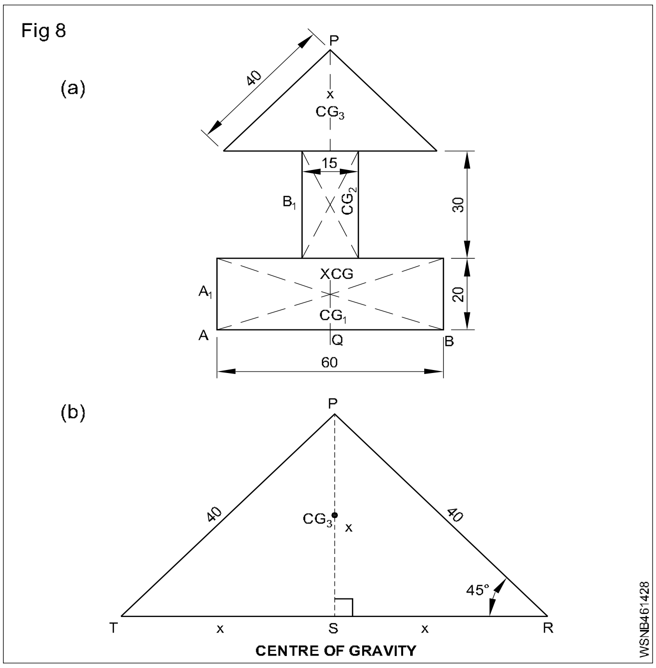
$$= \frac{1800(20) + 1125\left(\frac{170}{3}\right)}{1800 + 1125}$$

$$= \frac{36000 + 63753.75}{2925} = \frac{99753.75}{2925}$$

y = 34.10 mm

The CG is at a distance of 34.1mm from point F the point of reference in the line AB.

8 Find the CG of the lamina shown below.



CG is in PQ

CG_1, CG_2 and CG_3 - centres of centre of gravity.

Area of A_1 = 60 x 20 mm^2

=1200 mm^2

Distance of CG1, from AB $= \frac{20}{2}$ mm

= 10 mm

Area of B_1 = 30 x 15 mm^2

= 450 mm^2

Distance of CG_2 from AB $= 20 + \frac{30}{2}$ mm

= 20 + 15 mm

= 35 mm

Area of triangle $= \frac{1}{2} \times 40 \times 40 \text{ mm}^2$

= 800 mm^2

Δ PTR - Isosceles triangle

Draw perpendicular line PS on TR from P.

Δ PSR - right angled triangle

By applying Pythagoras theorem,

$x^2 + x^2 = 40^2$

$2x^2 = 1600$

$x^2 = 800$

$x = \sqrt{800}$

= 28.28 mm

Distance of CG_3 from TR $= \frac{x}{3} = \frac{28.28}{3}$ mm = 9.43 mm

Dist. Of CG_3 from AB = 20 + 30 + 9.43 mm = 59.43 mm

Total area =1200 + 450 + 800 mm^2 = 2450 mm^2

Distance from AB = Ymm

Taking moment at AB 2450 x y = 1200 x 10 + 450 x 35 + 800 x 9.43

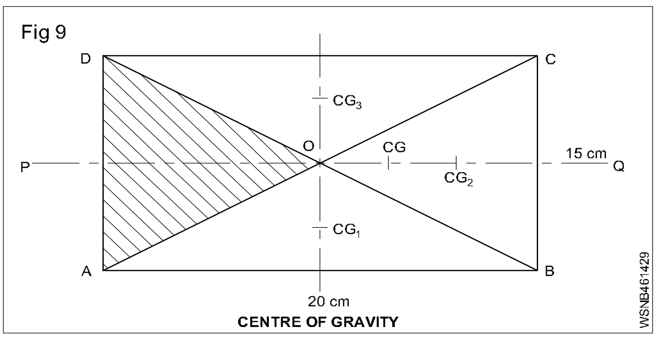
=12000 +15750 + 7544

=35294

$$y = \frac{35294}{2450} = 14.41 \text{ mm}$$

Distance of CG is on the line PQ from side AB = 14.41 mm.

9 A rectangular sheet of cardboard of uniform thickness measuring 20 cm by 15 cm is divided into four parts by drawing the diagonals. One of the triangles formed on a 15 cm side is removed. Find the position of the C.G. of the remainder.



ABCD - rectangle hard board

AC, BD - diagonals

O - meeting point

AOD - removed portion

Δ AOB = CG_1

Δ BOC = CG_2

Δ COD = CG_3

C.G = Centre of gravity of the hard board

Take CG is on PQ

Area of Δ AOB = 1/2 bh unit^2

= 1/2 x 20 x 7.5 cm^2

= 75 cm^2

Area of Δ BOC = 1/2 x 15 x 10 cm^2

= 75 cm^2

Area of Δ COD = 1/2 x 20 x 7.5 cm^2

= 75 cm^2

Total area = 75 + 75 + 75 cm^2

= 225 cm^2

Taking moment at side BC

Distance of CG_1 = $\frac{20}{2}$ in = 10 cm

Distance of CG_2 = $\frac{10}{3}$ in = 3.33 cm

Distance of CG_3 = $\frac{20}{2}$ in = 10 cm

Distance of C.G = x

225 x x = 75 x 10 + 75 x 3.33 + 75 x 10

= 750 + 249.75 + 750

= 1749.75

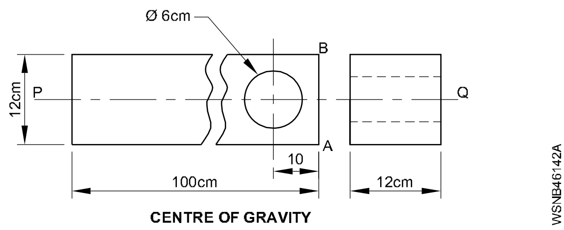
x = $\frac{1749.75}{225}$ = = 7.777

= 7.777

Distance of CG is on the line PQ from side BC=7.78cm

10 A steel rod 100x12x12cm has a hole dia 6cm drilled in it as shown in the figure. Find the position of the C.G. of the strip.

Fig 10



CENTRE OF GRAVITY

Volume of rod = a^{2}h unit^3

= 12 x 12 x 100 cm^3

= 14400 cm^3

Volume of circle = πr^{2}h unit^3

= π x 3 x 3 x 12 cm^3

= 339.3 cm^3

Remaining volume = 14400 - 339.8 cm^3

= 14060.7 cm^3

C.G is on PQ

CG in before drilling = $C.G_1$

CG in before drilling = $C.G_2$

CG in after drilling = C.G

Calculating the moment on side AB

Distance of CG_1 = $\frac{100}{2}$ = 50 cm

Distance of CG_2 = 10 cm

Distance of C.G = x

14060.7 x x + 339.3 x 10 =14400 x 50

14060.7x + 3393 = 720000

14060.7x = 720000 - 3393

= 716607

x = $\frac{716607}{14060.7}$

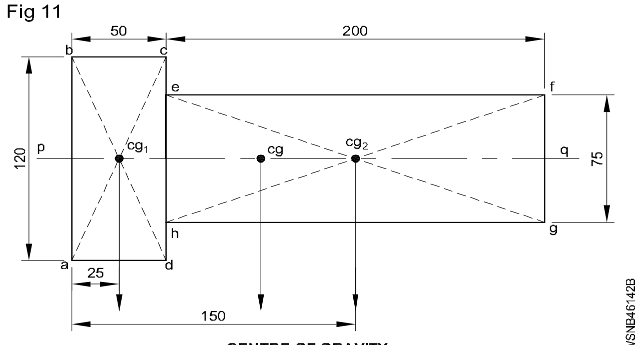
= 50.97 cm

C.G. of the strip = 50.97 cm from side AB.

11 Centre of gravity in a lamina (Area)

Find the position of c.g of the area shown in Fig11. (All dimensions are in mm.)

Fig 11



CENTRE OF GRAVITY

Solution

Taking moments of area about the line ab, we get the equation as below.

Moment of area abcd+moment of area efgh=moment of area of complete figure.

Now to calculate the areas

1 Area of abcd = 120 mm x 50 mm

= 6000 mm^2

Area of efgh = 200 x 75 mm^2

= 15000 mm^2

Total area = (6000+15000) mm^2

(abcd+efgh) = 21000 mm^2

2 (6000 m^2 x 25) + (15000 mm^2 + 150 mm)

= (21000mm^2) x (x mm)

150000mm^2 + 2250000mm^2 = (21000 mm^2) x (x mm)

240000mm^2 = (21000 mm^2) x (x mm)

$$\text{Therefore } x = \frac{2400000\,mm^2}{21000\,mm^2}$$

$$= \frac{2400}{21}\ mm$$

$$= \frac{800}{7}\ mm$$

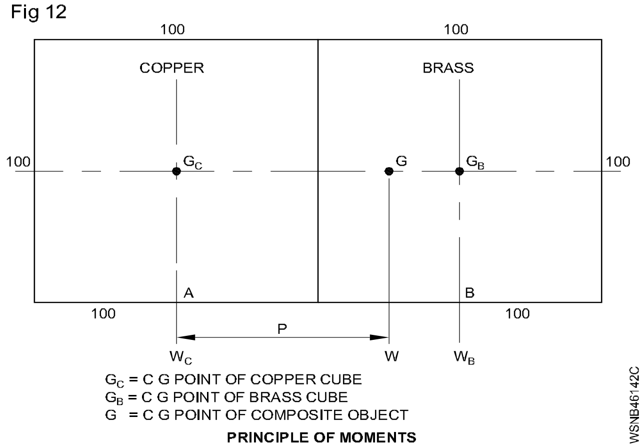
= 114.3 mm

Hence c.g. point of composite figure is 114.3 mm from A on the line ab.

12 Centre of gravity point of a composite body can be found out by using a variation of principle of moments.

Example (Fig 12)

Fig 12



Moment of part "A" about O+ Moment of part "B" about O=Moment of (A+B) about O.

The moment of the (A+B) acting through point G.

A copper cube of 100mm side is attached to brass cube of 100 mm side, as sketched in the figure. Calculate the position of c.g of composite object. Take densities of copper and brass as 8.9 gms/cm^3 and 8.5 gms/cm^3.

Solution

Volume of copper cube = 100 x 100 x 100 cm^3

= 10^6 mm^3

$$= \frac{10^6 mm^3}{10^3 mm^3}$$

= 1000 cm^3

Mass of copper cube = Volume x Density

= 1000 x 8.9

= 8900 gms

$$= \frac{8900}{1000}\ Kg$$

= 8.9 Kg.

(g=Acceleration due to gravity=10m/sec^2)

Weight of copper cube = 8.9 kg x 10 m/sec^2

Similarly = 89 N

$$\text{Weight of brass cube} = \frac{1000\ cm^3 \times 8.5 \times 10}{1000}$$

(Take g = 10m/sec^2) = 8.500 x 10 = 85N

Let cg of separate cubes be Gc and G_B as shown in figure.

The distance between Gc and GB=100mm or 0.1 m

Let c.g of the total object be at G which is 'P' meter to the right of G_C or (0.1-P) meter to the left of G_B.

Take moments about G

Clock moments = W_B x (0.1 - P)

= 85 x (0.1 - P))

= 8.5 - 85P

Anti clock moments = W_C x P

= 89 x P[Nm]

= 89P

By principle of moments

89P = 8.5 - 85P [Equating clock moments with anti-clock moments]

89P + 85P = 8.5

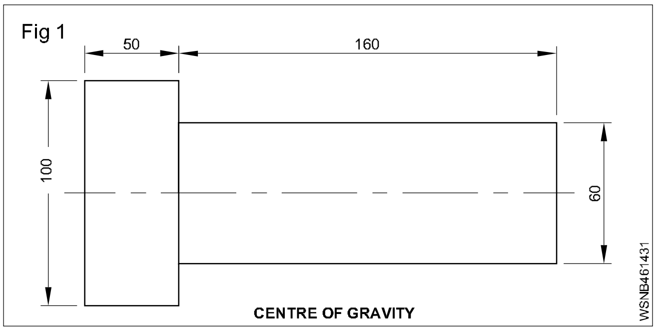
174P = 8.5

$$P = \frac{8.5}{174}\ \text{meter or 0.049 m or 49 mm}$$

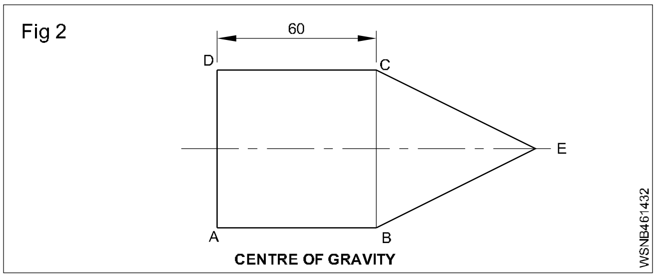
Centre of gravity of the composite object lies 49 mm from point of Gc. Hence it lies within copper cube.

Assignment A

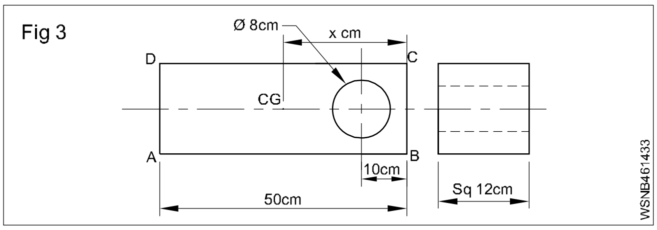
1 Find the position of centre of gravity of the figure shown. (All dimensions in mm)



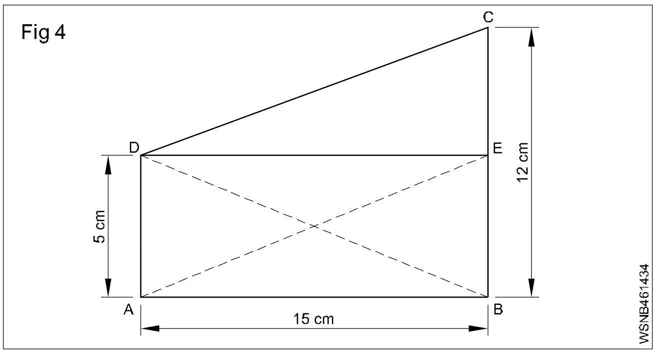
2 A lamina consists of a square of 60mm side, on one side of which an equilateral triangle is constructed. Find the position of centroid of the composite.



3 A steel strip 50x12x12cm has a hole of 8cm dia. drilled through it at a distance of 10cm from end. Find out the c.g of the strip.



4 Find out the C.G. of the four sided figure ABCD when ∠A = ∠B=90º and the side AB=15cm, BC=12cm and AD=5cm.



Assignment B

1 What is the centre of gravity of a semi-circle is at a perpendicular distance from its centre?

A $\frac{3r}{4\pi}$ B $\frac{4r}{3\pi}$

C $\frac{8r}{3}$ D $\frac{3r}{8}$

2 What is the centre of gravity of a hemisphere is at a distance from its space.

A B $\frac{3r}{8}$

C $\frac{4r}{8}$ D $\frac{5r}{8}$

3 What is the centre of gravity of a triangle is at the point where the medians of the triangle meet?

A $\frac{h}{2}$ B $\frac{h}{3}$

C $\frac{h}{4}$ D $\frac{h}{5}$

4 What is the centre of gravity of a right circular cone is at a distance from its base.

A $\frac{h}{2}$ B $\frac{h}{3}$

C $\frac{h}{4}$ D $\frac{h}{5}$

5 Centre of gravity is usually located where.

A more weight is concentrated

B less weight is concentrated

C less mass is concentrated

D more mass is concentrated

6 Centre of gravity of an object depends on it's.

A weight B mass

C density D shape

7 Point where whole weight of body acts vertically is called.

A centre of mass B mid point

C centre of gravity D none of above

8 A simple method to find centre of gravity of a body is usage of.

A stop watch

B plumbline

C pendulum

D screw gauge

9 If a material has no uniform density throughout the body, then the position of centroid and centre of mass are.

A identical

B not identical

C independent upon the density

D unpredictable

10 Which of the following laminas do not have centroid at its geometrical centre?

A Circle

B Equilateral triangle

C Right angled triangle

D Isosceles triangle

Key Answers

A

1 90.6 mm

2 44.3 mm

3 26.3 cm

4 The C.G. lies at a point at a distance of 4.49 cm, from the line AB and at a distance of 6.47 from the line BC.

B

1 B

2 B

3 B

4 C

5 D

6 B

7 C

8 B

9 B

10 C