Circle (Fig 1)

It is the path of a point which is always equal from its centre is called a circle.

r = radius of the circle

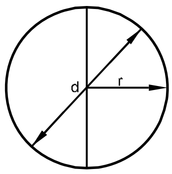
d = diametre of the circle

Area of the circle = πr^2

(or) = $\frac{\pi}{4} d^2$ unit^2

Circumference of the circle = $2\pi r$ (or) πd unit

Fig 1



PROPERTIES OF CIRCLE

WSNA320311

Sector of a circle (Fig 2)

The area bounded by an arc is called the sector of a circle. In the figure given ABC is the sector of a circle.

r = radius of the circle

θ = Angle of sector in degrees

Area of sector ABC

$$= \frac{\pi r^2 \times \theta}{360^0} \text{ unit}^2$$

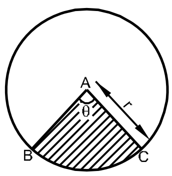
$$\text{Area of sector} = \frac{\text{Length of arc of sector} \times \text{radius}}{2} \text{ unit}^2$$

$$\text{Length of the arc } \ell = 2\pi r \times \frac{\theta}{360^0} \text{ unit}$$

Perimeter of the sector = $\ell + 2r$ unit

r = radius

Fig 2



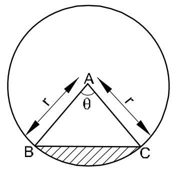
PROPERTIES OF CIRCLE

WSNA320312

Segment of a circle (Fig 3)

When a circle is divided into two by drawing a line, the bigger part is called segment of the circle and the smaller part is also called segment of the circle.

Fig 3



SEGMENT OF A CIRCLE

WSNA320313

Area of the smaller segment

= Area of the sector - Area of Δ ABC

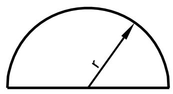
Area of the greater segment

= Area of the circle - Area of smaller segment

Semi Circle (Fig 4)

- A semi circle is a sector whose central angle is 180°.

Fig 4



SEMI CIRCLE

WSNA320314

- Length of arc of semi circle

$$\ell = 2\pi r \times \frac{180}{360} = 2\pi r \times \frac{1}{2}$$

$= \pi r$ unit

Area of semi circle = $\frac{\pi r^2}{2}$ unit^2

Perimeter of a semi circle $= \frac{2\pi r}{2} + 2r$

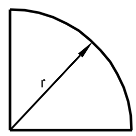
$= \pi r + 2r$

$= r(\pi + 2)$ unit

Quadrant of a circle (Fig 5)

- A quadrant of a circle is a sector whose central angle is 90°.

Fig 5



QUADRANT CIRCLE

WSNA320315

- Length and area of a quadrant of a circle

$$\ell = 2\pi r \times \frac{90}{360}$$

$$= 2\pi r \times \frac{1}{4}$$

$$= \frac{\pi r}{2}$$

Area of quadrant of a circle = $\frac{\pi r^2}{4}$ unit²

Perimeter of a quadrant = $\frac{2\pi r}{4} + 2r$

$$= \frac{\pi r}{2} + 2r$$

$$= r\left(\frac{\pi}{2} + 2\right) \text{ unit}$$

Examples :

1 Find the area of a sector of a circle whose radius is 14 cm and the length of the arc of the sector is 28 cm.

Radius of sector r = 14 cm

Length of arc of sector = 28 cm

Length of arc of sector (ℓ) = $\frac{\theta}{360} \times 2\pi r$ unit²

$$28 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 14 \text{ cm}^2$$

$$\theta = \frac{28 \times 360 \times 7}{2 \times 22 \times 14} = 114.55^0$$

∴ Angle of sector θ = 114.55^0

∴ Area of sector $= \frac{\theta}{360} \times \pi r^2$ unit²

$$= \frac{114.55}{360} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$$

= 196 cm²

Area of sector = 196 cm²

2 If the circumference of a circle is 44 cm, find its area. (Take $\pi = \frac{22}{7}$)

Solution

∴ Let (d) = diamter of circle

∴ Circumference of circle = πd

∴ 44 = π . d

$$d = \frac{44}{\pi} = 44 \div \pi$$

$$= 44 \div \frac{22}{7}$$

$$= 44 \times \frac{7}{22}$$

= 14 cm

∴ Diameter of circle (d) = 14 cm

∴ Area of circle $= \frac{\pi}{4} d^2$ unit²

$$= \pi \times \frac{1}{4} d^2$$

$$= \frac{22}{7} \times \frac{1}{4} \times 14 \times 14$$

= 154 cm²

Area of circle = 154 cm²

3 Find the remaining areas of circles of 10 cm dia after inscribing triangles of 5 cm base and 10 cm height.

Solution

(i) Area of the circle $= \frac{\pi}{4} d^2$

$$= \frac{22 \times 10 \times 10}{7 \times 4}$$

$$= \frac{550}{7}$$

(ii) Area of the triangle inscribed in this circle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

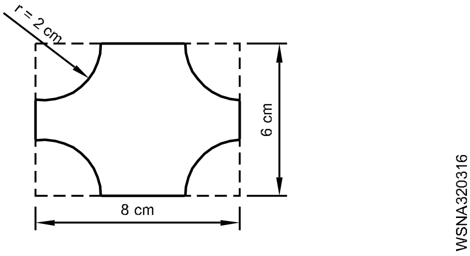
$$= \frac{10 \times 5}{2} = 25 \text{ sq.cm}$$

Remaining area $= \frac{550}{7} - 25$

Remaining area of circle $= 53\frac{4}{7}$ Sq.cm

4 A rectangular sheet of metal measures 8 cm and 6 cm. Four quadrants of circles each of radius 2 cm are cut away at corners. Find the area of the remaining portion.

Fig 6



Area of rectangular sheet = 8 x 6

= 48 cm^2

There are four quadrants of a circle, each of radius 2 cm cut away at the corners. Quadrant of circle means 1/4th of circle.

4 quadrant of circles = 4 x $\frac{1}{4}$ of circle = 1 circle

Area of 4 quadrant circles = Area of one circle

= πr^2

$= \frac{22}{7} \times 2 \times 2$

= 12.57 cm^2

Area of remaining portion =

Area of rectangular sheet - Area of four quadrant circles cut at corners.

= 48 - 12.57

= 35.428 cm^2

= say 35.43 cm^2

Area of remaining portion = 35.43 sq.cm

5 Find the perimeter of the given circular disc.

Sector :

r = 100 mm

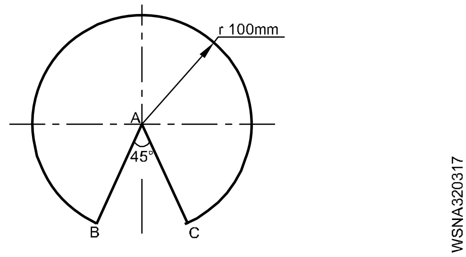
θ = 360° - 45° = 315°

$$\ell = \frac{\theta}{360} \times 2\pi r \text{ unit}$$

$$= \frac{315}{360} \times 2 \times \pi \times 100\text{mm}$$

ℓ = 550 mm

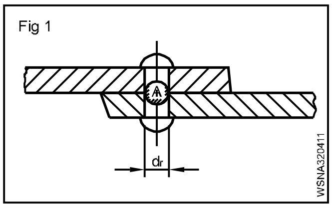
Fig 7



Perimeter of the given circular Disc = ℓ + 2r

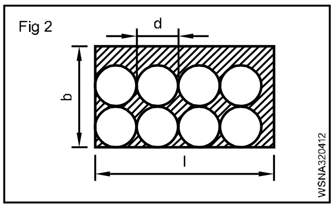
= 550 + 200 = 750 mm

Perimeter of the given circular Disc = 750 mm

1 

d_t = 21 mm

A_t = ______ mm²

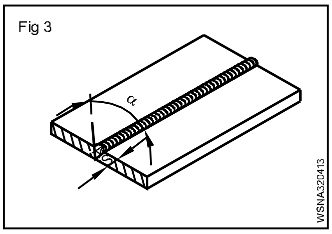
2 

l = 750 mm

b = 400 mm

d = 180 mm

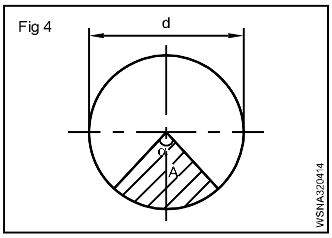
Area of sheet
= ________

3 

α = 60°

s = 9.2 mm

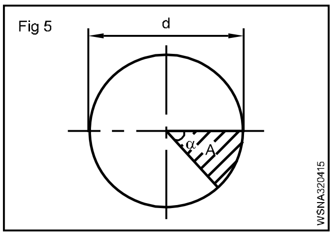
A of sector
= ________ mm²

4 

A = Area of sector
= 140 mm²

d of the circle
= 30 mm

α = ______°

5 

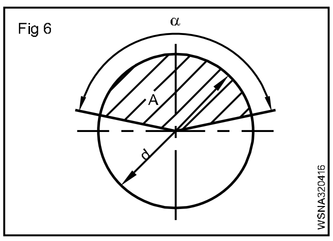
d = 380 mm

No. of sectors of equal area = 8

Area of each sector
= ______ mm²

α = ______ °

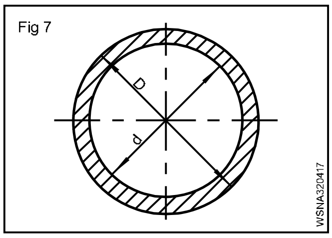
length of arc of each sector = ______ mm

6 

α = 160°

A = 0.893 m²

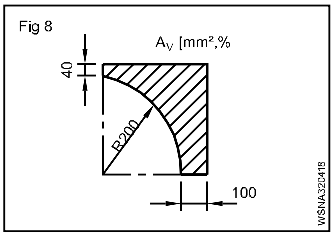
d = ______ mm

7 

D = 38 mm

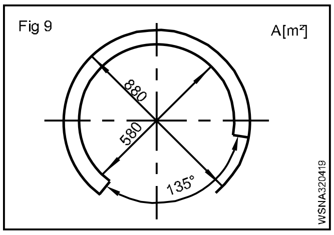
d = 32 mm

Cross sectional area
= _____ mm²

8 

Av (Area of shaded part) = ______ mm²

Av = % of (Area of rectangle) A_1

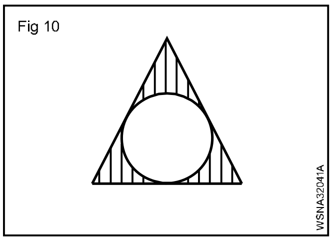
9 

D = 880 mm

d = 580 mm

Angle of cut off sector a = 135°

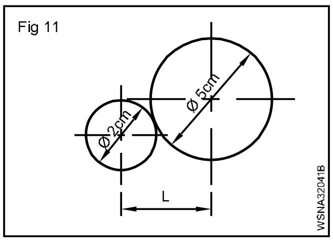
Area of the remaining portion, A
= _____ mm²

10 

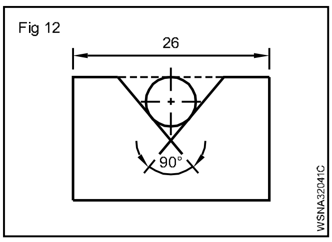
Equilateral triangle of side a = 6 cm

Radius of circle
= 1.732 cm

Shaded area
____

11 

Two plugs having diameters 2 cm and 5 cm are placed on a surface plate touching each other. calculate the distance 'L' in the figure.

12 

90° vee block is 26 mm wide at the top of the vee block. What dia. of soft when laid in the vee block will have its top surface just level with the top of the vee block.

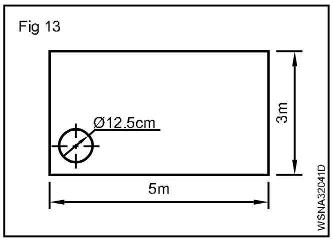
13 

Fig 13

From a sheet of 5m × 3m how many circular pieces of 12.5 cm dia can be cut.

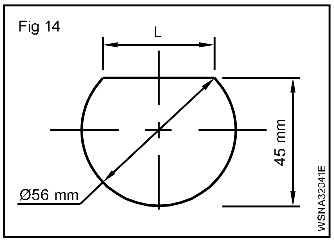
14 

Fig 14

Find out 'L' from the given sketch.

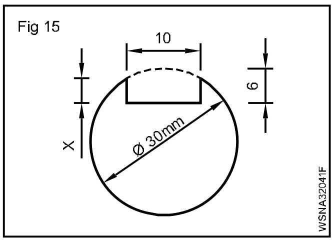
15 

Fig 15

Find the value of 'x' in the following fig.

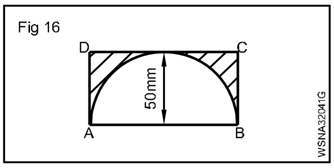
16 

Fig 16

Area of the shaded portion

______________ mm^2.

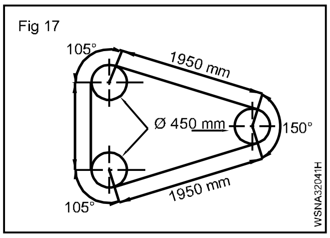
17 

Fig 17

The arrangement of a band saw blade is shown in the figure given below. Find out the length of the saw blade.

18 Calculate the area covered by 3 equal circles of radius 2.8 cm touches one another.

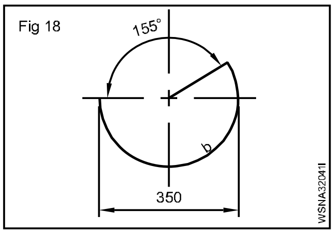
19 

Fig 18

α = 155°

d = 350 mm

b = ____mm

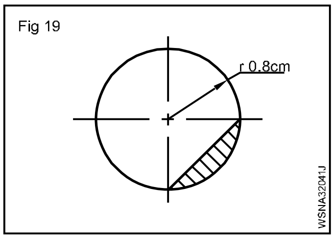
20 

Fig 19

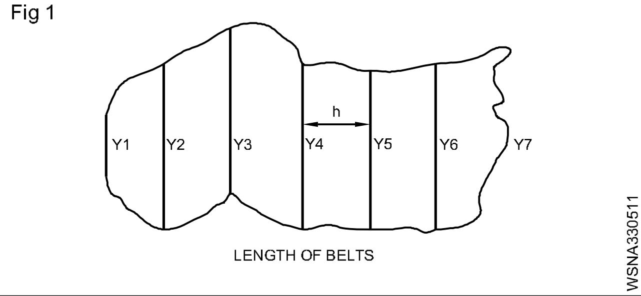
Find the area of shaded portion.

Area of irregular surfaces and application related to shop problems
Exercise 2.3.07

Area of irregular surface

Surface area of irregular figures can be obtained by applying either i) simpson's rule or ii) trapezoidal rule. Area found by simpson's rule is more accurate than trapezoidal rule. However accurate area can be obtained if the number of ordinates are more i.e interval between ordinates is so small as possible. (Fig 1)

Fig 1



i Area as per simpson's rule

$$\text{Area} = \frac{h}{3}\left[y_1 + y_7 + 4(y_2 + y_4 + y_6) + 2(y_3 + y_5)\right]$$

where

h = interval between ordinates

ii Area as per trapezoidal rule

$$\text{Area} = \left[\frac{h}{2}\text{ (first ordinate + last ordinate) + sum of remaining ordinate}\right]$$

Calculate the area enclosed between the chain line, the edge and the end offsets by

The offsets were taken from a chain line to a edge.

Distance (M)	0	5	10	15	20	25	30	35
Off set (M)	4	3	2	5	1	2	3	5

(a) Simpson's rule **(b) Trapezoidal rule**

(a) Simpson's rule

$$A = \frac{h}{3}\left[y_1 + y_8 + 4(y_2 + y_4 + y_6) + 2(y_3 + y_{5+}Y_7)\right] \text{ unit}^2$$

$$A = \frac{5}{3}\left[4 + 5 + 4(3 + 5 + 2) + 2(2 + 1 + 3)\right] \text{ m}^2$$

= 101.7 m² .

(b) Trapezoidal rule

$$A = \frac{h}{3}\left[y_1 + y_8 + 2(y_2 + y_3 + y_4 + y_5 + y_{6+}Y_7)\right] \text{ unit}^2$$

$$A = \frac{5}{2}\left[4 + 5 + 2(3 + 2 + 5 + 1 + 2 + 3)\right] \text{ m}^2$$

$$A = \frac{5}{2} \times 41 \text{ m}^2$$

= 102.5 m²

Calculation of the area of an irregular surface

In this Calculation the area of an irregular surface may be determined as follows.

In this method of calculation a chain line known as base line to be laid through the centre of the area of the surface.

The offset are taken to the boundary points in the order of their chainages on both the sides of the base line.

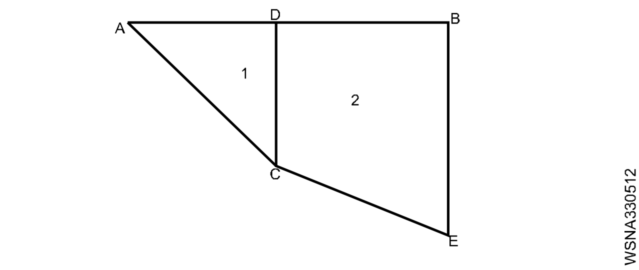
The chain line and offsets are noted down.

With reference to the notes the boundary points are plotted and the area to be divided into number of triangles and trapezium according to the shape.

Example

Now apply the geometrical formulae for calculating the according to the shape of the figures. (Fig 2)

Fig 2



Chainline = AB

Offsets = C,E

1 Area of triangle

½ x base x height

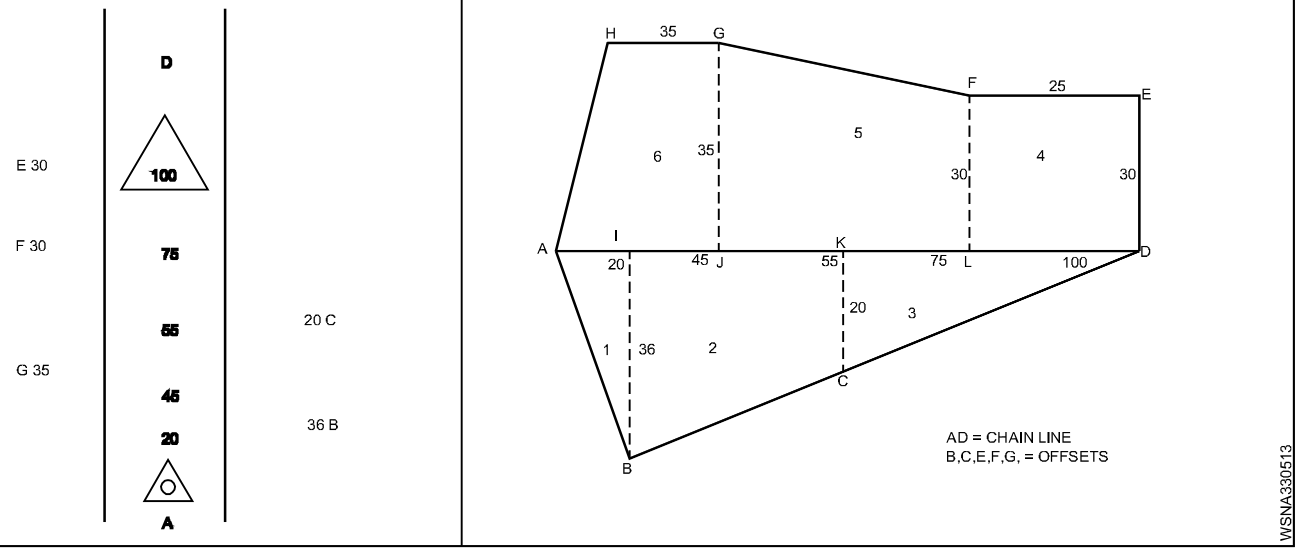
2 Area of trapezium

$$\frac{\text{base }(a+b)}{2} \times \text{height}$$

Example

Plot the following details of a field and calculate its area all measurements are in metres (Fig 3)

Fig 3



Serial No. 1 In ΔABI

Chainage in metres 0 and 20m.

Offsets in metres 0 and 36m.

In Δ ABI

Area = ½ x base x height

=1/2 x 20 x 36

=360 sq.m

Sl. No. 2

Area of trapezium IBCK

Chainage in metres = 20m and 55m = 35m

Offsets in metres 36m and 20m = 28m

$$= \frac{(a+b)}{2} \times \text{height} = \left[\frac{36+20}{2} \times 35\right]$$

= 28 x 35 = 980 sq.m

Sl. No. 3

Area of triangle KCD $= \frac{1}{2}$ x b x h = $\frac{1}{2} \times 20 \times 45$

= 45m x 10m = 450 Sq.m

Sl. No. 4

Area of rectangle DEFL = 25 x 30 = 750 sq.m

Sl. No. 5 (LFGH)

Area of Trapezium LFGJ = $\frac{(a+b)}{2} \times \text{height} = \left[\frac{30+35}{2} \times 30\right]$

= 32.5m x 30m = 975 sq.m

Sl. No. 6

Area of trapezium AJGH $= \frac{35+45}{2} \times 35 = \frac{80}{2} \times 35$

= 40 x 35 = 1400 sq.m

S. No.	Figure	Chainline in metres	Base in Metres	Offsets in metres	Mean offsets in metres	Area in square Metres		Remarks
						+ve	-ve	
1	2	3	4	5	6	7	8	9
1	ΔABI	0 and 20	20	0 and 36	18	360	--	
2	Trapezium IBCK	20 and 55	35	36 and 20	28	980	--	
3	ΔKCD	55 and 100	45	0 and 20	10	450	--	
4	Rectangle DEFL	100 and 75	25	0 and 30	15	750	--	
5	Trapezium LFGJ	75 and 45	30	30 and 35	32.50	975	--	
6	Trapezium JGHA	45 and 0	45	45 and 35	40	1400	--	
					Total	4915		

Assignment

Calculate the area of the irregular surfaces given below.

Fig 1

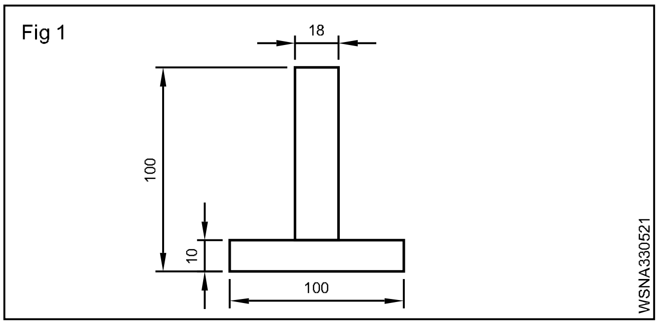


Fig 2

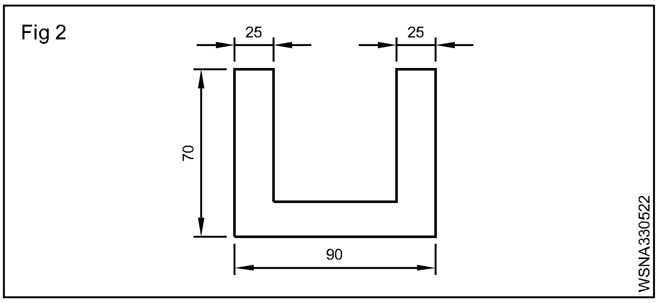


Fig 3

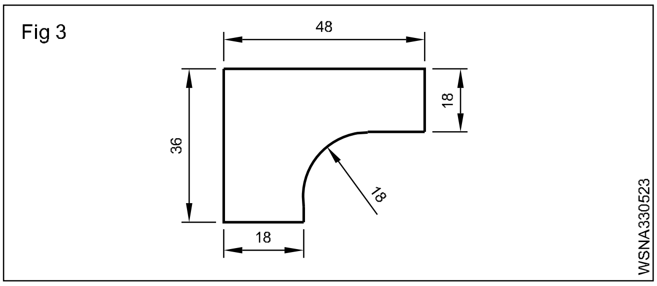


Fig 4

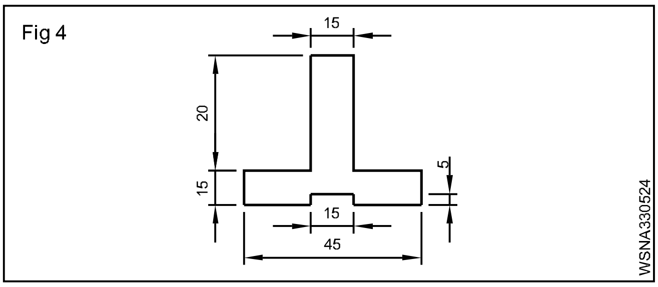


Fig 5

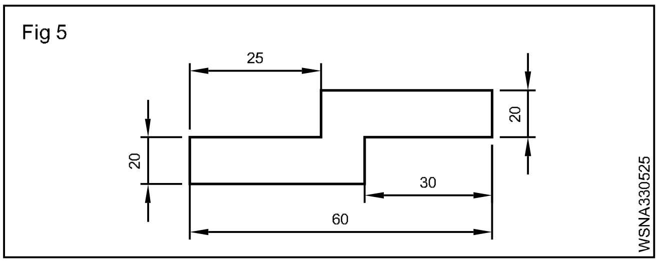


Fig 6

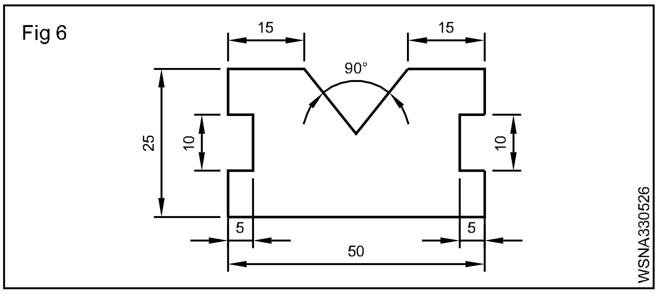


Fig 7

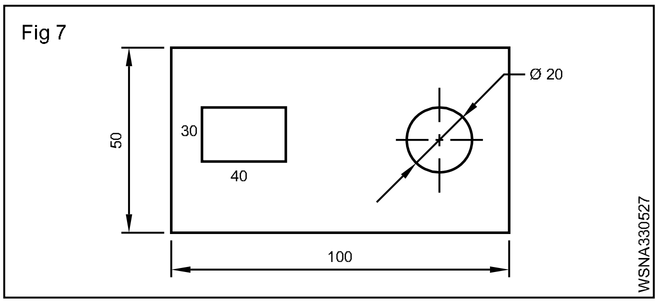
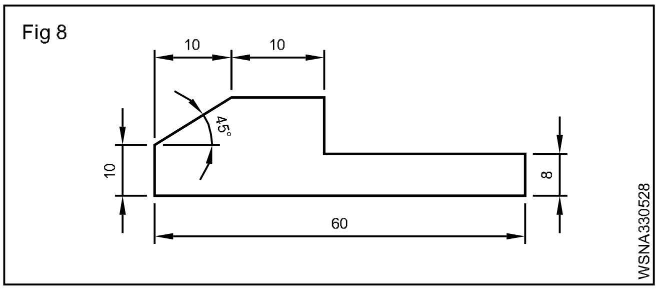


Fig 8



Note : All dimension are in mm.