# Exercise 2.4.08

## Introduction

Algebra is a form of mathematics in which letters may be used in place of unknown. In this mathematics numbers are also used in addition to the letters and the value of number depends upon its place. For example in $3x$ and $x^3$, the place of $x$ is different. In $3x$ =3 is multiplied with $x$, whereas in $x^3$ - 3 is an Index of $x$.

## Positive and negative numbers

Positive numbers have a + sign in front of them, and negative numbers have – sign in front of them. The same applies to letters also.

**Example** $+x$, – y.

+8 or simply 8 positive number.

– 8 negative number.

## Addition and subtraction

Two positive numbers are added, by adding their absolute magnitude and prefix the plus sign.

To add two negative numbers, add their absolute magnitude and prefix the minus sign.

To add a positive and a negative number, obtain the difference of their absolute magnitudes and prefix the sign of the number having the greater magnitude.

$+7 + 22 = +29$

$-8 - 34 = -42$

$-27 + 19 = -8$

$44 + (-18) = +26$

$37 + (-52) = -15$

## Multiplication of positive and negative numbers

The product of two numbers having like signs is positive and the product of two numbers with unlike signs is negative. Note that, where both the numbers are negative, their product is positive.

**Ex.**

$-20 \times -3 = 60$

$5 \times 8 = 40$

$4 \times -13 = -52$

$-5 \times 12 = -60$

## Division

The number that is divided is the dividend, the number by which we are dividing is the divisor and the answer is the quotient. If the signs of the dividend and the divisor are the same then the quotient will have a + sign. If they are unlike then the quotient will have a negative sign.

$$\frac{+28}{+4} = +7$$

$$\frac{+56}{-4} = -14$$

$$\frac{-72}{+9} = -8$$

$$\frac{-96}{-6} = +16$$

**When an expression contains addition, subtraction, multiplication and division, perform the multiplication and division operations first and then do the addition and subtraction.**

**Example**

$12 \times 8 - 6 + 4 \times 12 = 96 - 6 + 48 = 138$

$102 \div 6 - 6 \times 2 + 3 = 17 - 12 + 3 = 8$

## Parantheses and grouping symbols

( ) Brackets

{ } Braces

$7 + (6-2) = 7 + 4 = 11$

$6 \times (8-5) = 6 \times 3 = 18$

## Parentheses

These are symbols that indicate that certain addition and subtraction operations should precede multiplication and division. They indicate that the operations within them should be carried out completely before the remaining operations are performed. After completing the grouping, the symbols may be removed.

In an expression where grouping symbols immediately preceded or followed by a number but with the signs of operation omitted, it is understood, that multiplication should be performed.

Grouping symbols are used when subtraction and multiplication of negative numbers is done.

To remove grouping symbols which are preceded by negative signs, the signs of all terms inside the grouping symbols must be changed (from plus to minus and minus to plus).

Parentheses which are preceded by a plus sign may be removed without changing the signs of the terms within the parentheses.

When one set of grouping symbols is included within another set, remove the innermost set first.

When several terms connected by + or – signs contain a common quantity, this common quantity may be placed in front of a parentheses.

$8 + 6(4-1) = 8 + 6 \times 3 = 26$

$(6+2)(9-5) = 8 \times 4 = 32$

Plus 4 less negative 7 is written as 4 – (–7).

Plus 4 times negative 7 is written as 4(–7).

$4 - (-7) = 4 + 7 = 11$

$8 - (7-4) = 8 - 3 = 5$

$3 + (-8) = 3 - 8 = -5$

$7 + (4 - 19) = 7 + (-15) = 7 - 15 = -8$

$3\{40 + (7 + 5)(8-2)\}$

$= 3\{40 + 12 \times 6\}$

$= 3 \times 112 = 336.$

$8x + 12$ - quantity 4 may be factored out giving the expression $8x + 12$ as $4(2x + 3)$.

The innermost set in a grouping symbols of an expression is to be simplified first.

**Algebraic symbols and simple equations**

**Algebraic symbol**

An unknown numerical value of a quantity is represented by a letter which is the algebraic symbol.

**Factor**

A factor is any one of the numbers or letters or groups which when multiplied together give the expression. Factors of 12 are 4 and 3 or 6 and 2 or 12 and 1.

$8x + 12$ is the expression and this may be written as $4(2x + 3)$, 4 and $(2x + 3)$ are the factors.

**Algebraic terms**

If an expression contains two or more parts separated by either + or –, each part is known as the term.

$y - 5x$ is the expression. y and $-5x$ are the terms.

The sign must precede the term.

**Kinds of terms:**

**1. Like terms**

a) 13a, 15a, 19a, -12a, -18a

b) $5xy$, $11xy$, $-xy$, $-14xy$

c) $27m^2$, $25m^2$, $-3m^2$, $11m^2$

**2. Unlike terms**

a) 3ac, - 4b, $8x$, 3yz

b) $2xy$, $y^2$, $a^2b$, $xz$, 3bc

c) $13m^2n$, $3mn^2$, $14lm^2$, $15a^2b$, 5lm

**Examples :**

1) Add 7a, - 2a, a, 3a

$7a + (-2a) + (a) + 3a$

$7a - 2a + a + 3a$

$= 11a - 2a$

$= 9a$

2) Add $25xy$, $+ 2xy$, $- 6xy$, $- 3xy$

$25xy + 2xy + (-6xy) + (-3xy)$

$= 27xy - 9xy$

$= 18xy$

3) Add 9m, + 4m, - 2

$9m + 4m + (-2)$

$9m + 4m - 2$

$= 13m - 2$

4) Add $5a^3$, $+ 12b^3$, $- c^3$, $+ a^3$, $- 4b^3$, + 3

$5a^3 + 12b^3 + (-c^3) + a^3 + (-4b^3) + 3$

$= 6a^3 + 8b^3 - c^3 + 3$

**Coefficient**

When an expression is formed into factors whose product is the expression, then each factor is the coefficient of the remaining factors.

$48x = 4 \times 12 \times x$

4 is the coefficient of $12x$ . $x$ is the coefficient of 48.

**Equation**

It is a statement of equality between numbers or numbers and algebraic symbols.

$12 = 6 \times 2$, $13 + 5 = 18$.

$2x + 9 = 5$, $y - 7 = 4y + 5$.

**Simple equation**

Equations involving algebraic symbols to the first power are simple equations.

$2x + 4 = 10$.  $4x + 12 = 14$.

**Addition and subtraction**

Quantities with algebraic symbols are added or subtracted by considering those terms involving same symbols and powers.

Example

1. $10x + 14 - 7y^2 - 11a + 2x - 4 - 3y^2 - 4a + 8$

   $= 10x + 2x - 7y^2 - 3y^2 - 11a - 4a + 14 - 4 + 8$

   $= 12x - 10y^2 - 15a + 18$

2. $2x = 10$, $2x + 6 = 10 + 6$

3. $y + 12 = 20$, $y + 12 - 8 = 20 - 8$

4. $x + 10 = 12$,

   $x + 10 - 10 = 12 - 10$

5. $3x = 6$, $2 \times 3x = 2 \times 6$, $6x = 12$

6. $5y = 20$, $\frac{5y}{5} = \frac{20}{5}$.

The same number may be added or subtracted to both members of an equation without changing its equality.

Each member of an equation may be multiplied or divided by the same number or symbol without changing its equality.

The equality of an equation is not altered when the numbers or symbols are added or subtracted from both sides. Multiplication and division by the same numbers or symbols on both sides also will not affect the equality.

**Transposition of the terms of the equations**

= equals to

+ plus

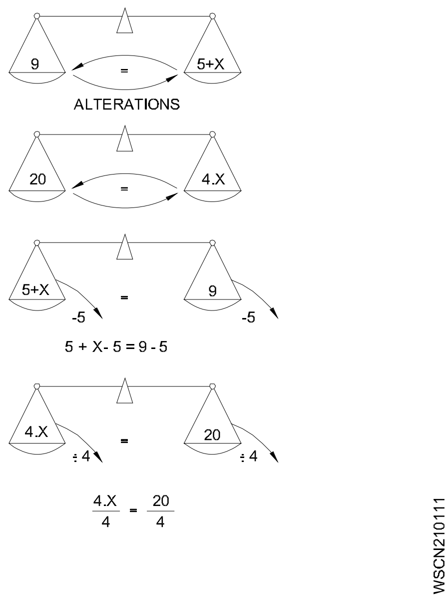
– minus

x multiply

÷ divided by

**Concept of equality** (Fig 1)

Fig 1




An equation can be compared to a pair of scales which always remain in equilibrium. The two sides of the equation can fully be transposed. $9 = 5 + x$ may also be written as $5 + x = 9$.

We must always perform the same operation on both sides of the equation to keep the equilibrium. Add or subtract the same amount from both sides. $5 + x = 9$ By adding 3 on both sides, the equation becomes $5 + x + 3 = 9 + 3$ or $x + 8 = 12$.

$5 + x = 9$ Subtract 5 from both sides then $5 + x – 5 = 9 – 5$.

$x = 4$.

5 is transposed from left side to the right side by changing its sign from + to –.

$\frac{x}{4} = 20$. Multiply both sides by 4. Then $\frac{x}{4} \times 4 = 20 \times 4$.

$x = 80$,

$5x = 25$.

Divide both sides by 5 then $\frac{5x}{5} = \frac{25}{5}$

$x = 5$.

When transposing numbers or letter symbols from one side to the other side multiplication becomes division and the division becomes multiplication.

**The equality of an equation remains unchanged when both sides of the equation are treated in the same way. When transposing from one side to the other side,**

**a plus quantity becomes minus quantity.**

**a minus quantity becomes a plus quantity**

**a multiplication becomes a division**

**a division becomes a multiplication.**

**To solve simple equations isolate the unknown quantity which is to be found on the left side of the equation.**

**Example**

- Solve for x if $4x = 3(35 – x)$

  $4x = 105 – 3x$ (brackets removed)

  $4x + 3x = 105$ (By transposing $–3x$ on the right side to the left side)

  $7x = 105$

  $x = 15$ (dividing both sides by 7)

## Assignment

**ADD**

1 $14f - 2f + 5f - 7f + 9f$

2 $3xy + 5xy - 2xy + 8xy - 4xy$

3 $7a^2 - 5a^2 + a^2 + 3a^2$

4 $3m^2n - 2m^2n + 4m^2n - m^2n + 7m^2n$

5 $18 + 13x^2 - 13 + 2x^2 - 15x^2$

6 $17xy - 4xy + 13 - xy - 6$

7 $6l^2m + 3lm^2 - 2l^2m - 17lm^2 + 1$

8 $3a^2b - 2ab - 2a^2b - 3ab - 2a^2b + ab$

9 $2a + a + 3a + 6a - 5b$

10 $8c + 5c + 3c + 2c$

11 $14d + 3d + 25e + 2e$

12 $5p + 3r - r - 2p$

13 $8t + 12u - t - 14u$

14 $x - z + y + z$

15 $15a + 13a - 37a$

16 $17a - 4b - 7a + 3b$

17 $9c - 15e + 4c + 3e$

18 $13f + 40g - 16f + 7f + 2g - 17g$

19 $30x + 45y - 17x - 16y$

20 $14.5k + 33.2 - 18.4k + 12.3$

21 $8a + 3c - 6b - 5c + 4a + 8b$

22 $27i + 17k - 5l + 12i - 31k + 19l$

23 $230m + 472P - 320n - 75m + 180n - 141p$

24 $17.5P + 12.4x - 29.2r - 7.6x - 12.1p + 31.4r$

25 $230m + 420s + 370y + 225m - 510y - 110s$

**Calculations involving powers**

**Power : Concept**

a.a.a... upto n times is = $a^n$

a is the base, n is the exponent.

When a number, say 2 is multiplied by itself 4 times, we write it as $2^4$ (two to the power of 4) and it is equal to 2 x 2 x 2 x 2 = 16.

The exponent denotes how many times the base number is multiplied by itself.

Powers with a positive base have a positive result.

Powers with a negative base and with an exponent that is even will have a positive result.

The sign

$(+a)^n = a^n$

$(-a)^{2n} = a^{2n}$

$(2)^2 = 2 \times 2 = 4$ and

$(-2)^2 = -2 \times -2 = +4$ but

$(-2)^3 = -2 \times -2 \times -2 = -8$

**Addition and subtraction of powers**

Powers with the same base and exponents can be added or subtracted by addition or subtraction of the coefficients.

$x.a^n + y.a^n = a^n(x + y)$

$x.a^n - y.a^n = a^n(x - y)$

Ex .$4x^2 + x^2 - 3x^2 = x^2(4 + 1 - 3) = 2x^2$.

**Multiplication**

Powers with the same bases are multiplied by involving the common base raised to the power of sum of the exponents.

$a^m \times a^n = a^{m+n}$.

$2^3 \times 2^2 = 2^{3+2} = 2^5$

$(2 \times 2 \times 2) \times (2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

8 x 4 = 32.

Powers with the same exponent of different base numbers are multiplied by involving the product of the base numbers raised to the common exponent.

$a^n \times b^n = (a \times b)^n$

$2^2 \times 3^2 = (2 \times 3)^2$

2 x 2 x 3 x 3 = 6 x 6 = 36

**Division**

Powers with like bases are divided by involving the base raised to the difference between the exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$$

$$\frac{2 \times 2 \times 2}{2 \times 2} = \frac{8}{4} = 2$$

Powers with the same exponents are divided by involving the quotient of the bases by the common exponent.

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2 = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$$

Only like powers can be added or subtracted.

**Examples**

(The exponent 1 is usually not written.)

$a^1 = a$

$2^1 = 2$

$2a^2 + 3a^2 = 5a^2$

(Any number raised to the power of 0 is 1.)

$a^0 = 1$

$2^0 = 1$

A number raised to a negative power corresponds to its reciprocal with the exponent's sign changed to +.

$$a^{-n} = \frac{1}{a^n}$$

$$2^{-2} = \frac{1}{2^2}$$

Powers are involved by multiplying the exponents.

$(a^n)^m = a^{nm}$

$(2^2)^3 = 2^{2.3} = 2^6$

Powers can be transposed without affecting the result.

$$\left(a^n\right)^m = \left(a^m\right)^n$$

$(2^2)^3 = (2^3)^2$

(2 x 2) x (2 x 2) x (2 x 2) = (2 x 2 x 2) (2 x 2 x 2)

4 x 4 x 4 = 64

8 x 8 = 64

A mixed number raised to a power is first converted into an improper fraction and then the result is evaluated.

$$\left(1\frac{3}{4}\right)^2 = \left(\frac{7}{4}\right)^2$$

$$= \frac{7}{4} \times \frac{7}{4} = \frac{49}{16}$$

**Indices**

- The indices are added in multiplication

  $a^m \times a^n = a^{m+n}$.

- The indices are subtracted in division

  $$\frac{a^m}{a^n} = a^{m-n}$$

- In case of index of an index, both the indices are multiplied mutually

  $[a^m]^n = a^{m.n}$

- A fractional index shows root of a number

  $a^{1/m} = \sqrt[m]{a}$

- In case of an index having minus sign, the sign can be changed by taking the number from numerator to denominator or vice versa

  $$a^{-m} = \frac{1}{a^m}$$

  and $$\frac{1}{a^{-m}} = a^m$$

- If an index contains both the numerator and denominator then it means that the number has 'index' as well as 'root'.

  $a^{m/n} = \sqrt[n]{a^m}$

**Basic problem**

**Addition**

1. 8a + 12b - a - 14b

   = 8a - a + 12b - 14b

   = 7a - 2b

2. 14a + 3a + 25b + 2b + b

   = 17a + 28b

3. (2a + 3b - c) + (4a - b - c) + (a - 8)

   2a + 3b - c + 0

   4a - b - c + 0

   a + 0 + 0 - 8

   ______________

   7a + 2b - 2c - 8

   ______________

4. Add : $(3x + 3z)$ ; $(5x - 4y)$; $(9y - 3z)$

   $3x + 0 + 3z$

   $5x - 4y + 0$

   $0 + 9y - 3z$

   ______________

   $8x + 5y$

   ______________

5. $5x^2y + 3xy^2 + 8x^2y + 7xy^2$

   $= 5x^2y + 8x^2y + 3xy^2 + 7xy^2$

   $= 13x^2y + 10xy^2$

**Subtraction**

1. $38xy - 15xy = 23xy$

2. Subtract $3xy$ from $-4xy$

   $-4xy$

   (-) $+3xy$

   ______

   $-7xy$

   ______

3. Subtract $5x$ from $12x$

   $= 12x - (5x)$

   $= 12x - 5x$

   $= 7x$

4. Subtract $18x$ from $7x$

   $= 7x -( 18x)$

   $= 7x - 18x$

   $= -11x$

5. Subtract $2x^2 - 3y^2$ from $3x^2 + 2y^2$

   $3x^2 + 2y^2$

   $2x^2 - 3y^2$

   ______________

   $x^2 + 5y^2$

   ______________

6. Subtract $3x - 2y$ from $4y - 2x$

   $= (4y - 2x) - (3x - 2y)$

   $= 4y - 2x - 3x + 2y$

   $= 6y - 5x$

**Multiplication**

1. $\underset{(-)}{-4x^2} \times \underset{(+)}{8x^5} = -4 \times 8x^{2+5}$

   $= -32x^7$

2. $(3d^2 - 2d)\,3d$

   $= 9d^3 - 6d^2$

3. $(5x + 3y)(5x - 3y)$

   $= (5x)^2 - (3y)^2$

   $= 5x \times 5x - 3y \times 3y$

   $= 25x^2 - 9y^2$

4. $5x^2y \times 8x^5y^3$

$= 40x^7y^4$

5. (2a+b) (a+2b)

$= 2a^2 + 4ab + ab + 2b^2$

$= 2a^2 + 2b^2 + 5ab$

6. $8a^3b^5c^{-5} \times 3a^2b^{-5}c^5$

$= 24a^5$

**Division**

1. $\frac{12x^3y^2}{4x^2y} = 3xy$

2. $\frac{15y^{15}}{15y^5} = y^{10}$

3. $9c^5d^3 \div c^2d^2$

$= 9c^3d$

4. $\frac{5a+20}{7a+28} = \frac{5(a+4)}{7(a+4)} = \frac{5}{7}$

5. $\frac{3a^2 \times 4a \times 5a^3}{6a^4 \times 10a}$

$= \frac{60a^6}{60a^5} = a$

6. $-25a^{15} \div -5a^{-8}$

$= \frac{-25a^{15}}{-5a^{-8}}$

$= 5a^{15}a^8 = 5a^{23}$

7. $4x^2y \div 2y$

$= \frac{4x^2y}{2y} = 2x^2$

8. $3x^2y^3 \div -6x^5y$

$= \frac{3x^2y^3}{-6x^5y} = -\frac{y^2}{2x^3}$

9. $3x^3y^2 \div xy$

$= \frac{3x^3y^2}{xy} = 3x^2y$

10. Divide $45a^2b^2c$ by $9a^2c$

$= \frac{45a^2b^2c}{9a^2c}$

$= 5b^2$

## Algebraic Formulae

| | | | |
|---|---|---|---|
| 1 | $(a + b)^2$ | = | $a^2 + b^2 + 2ab$ |
| 2 | $(a - b)^2$ | = | $a^2 + b^2 - 2ab$ |
| 3 | $(a + b)^2$ | = | $(a - b)^2 + 4ab$ |
| 4 | $(a - b)^2$ | = | $(a + b)^2 - 4ab$ ; $(a + b)^2 - (a - b)^2 = 4ab$ |
| 5 | $a^2 + b^2$ | = | $(a + b)^2 - 2ab = (a - b)^2 + 2ab$ |
| 6 | $a^2 - b^2$ | = | $(a + b)(a - b)$ |
| 7 | $a^3 + b^3$ | = | $(a + b)(a^2 + b^2 - ab)$ |
| 8 | $a^3 - b^3$ | = | $(a - b)(a^2 + b^2 + ab)$ |
| 9 | $(a + b)^3$ | = | $a^3 + b^3 + 3ab(a + b)$ |
| 10 | $(a - b)^3$ | = | $a^3 - b^3 - 3ab(a - b)$ |
| 11 | $(a + b + c)^2$ | = | $a^2 + b^2 + c^2 + 2(ab + bc + ca)$ |
| 12 | $a^4 - b^4$ | = | $(a^2 + b^2)(a + b)(a - b)$ |

**Examples**

1) **If x + y = 9 and xy = 20**

Find i) $x^2 + y^2$ ii) $x - y$ iii) $x^2 - y^2$

iv) $x^3 + y^3$ v) $x^3 - y^3$ vi) x and y

i) $\mathbf{(a + b)^2 = a^2 + b^2 + 2ab}$

$(x + y)^2 = x^2 + y^2 + 2xy$

$(9)^2 = x^2 + y^2 + 2(20)$

$81 = x^2 + y^2 + 40$

$x^2 + y^2 = 81 - 40$

$x^2 + y^2 = 41$

ii) $\mathbf{(a - b)^2 = (a + b)^2 - 4ab}$

$(x - y)^2 = (x + y)^2 - 4xy$

$= (9)^2 - 4(20)$

$= 81 - 80$

$= 1$

$x - y = \sqrt{1} = 1$

iii) $\mathbf{a^2 - b^2 = (a + b)(a - b)}$

$x^2 - y^2 = (x + y)(x - y)$

$= 9 \times 1$

$x^2 - y^2 = 9$

iv) $\mathbf{a^3 + b^3 = (a + b)(a^2 + b^2 - ab)}$

$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$= 9(41 - 20)$

$= 9 \times 21$

$x^3 + y^3 = 189$

v) **$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$**

$x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

$= 1 (41 + 20)$

$= 1 \times 61$

$= 61$

$x^3 - y^3 = 61$

vi)

$x + y = 9$

$x - y = 1$

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$2x = 10$

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$x = \frac{10}{2} = 5$

If x = 5, 5 + y = 9

y = 9 - 5 = 4

**x = 5; y = 4**

2. Solve $(x + 5)^2 - (x - 5)^2$

If x + 5 = a and x - 5 = b

$a^2 - b^2 = (a + b)(a - b)$

$(x + 5)^2 - (x - 5)^2 = [(x + 5) + (x - 5)]\,[(x + 5) - (x - 5)]$

$= (x + 5 + x - 5)(x + 5 - x + 5)$

$= (2x)(10)$

$= 20x$

3. If (x - y) = 4 and xy = 12, find the value of $(x^2 + y^2)$

$(x - y)^2 = x^2 + y^2 - 2xy$

$(4)^2 = x^2 + y^2 - 2 \times 12$

$16 = x^2 + y^2 - 24$

$x^2 + y^2 - 24 = 16$

$x^2 + y^2 = 16 + 24$

$x^2 + y^2 = 40$

4. If x - y = 7 and xy = 60 then find the value of $x^4 + y^4$

$(x - y)^2 = x^2 + y^2 - 2xy = 7^2$

$x^2 + y^2 - 2 \times 60 = 49$

$x^2 + y^2 = 169$

$(x^2 + y^2)^2 = (169)^2$ (take square on both side)

$x^4 + y^4 + 2x^2y^2 = (169)^2$

$x^4 + y^4 + 2(xy)^2 = 28561$

$x^4 + y^4 + 2(60)^2 = 28561$

$x^4 + y^4 + 2(3600) = 28561$

$x^4 + y^4 + 7200 = 28561$

$x^4 + y^4 = 28561 - 7200$

$x^4 + y^4 = 21361$

5. $x + y = \sqrt{5}$; $x - y = \sqrt{3}$ Find the value of $8xy(x^2 + y^2)$

$x + y = \sqrt{5}$; $x - y = \sqrt{3}$ (take square on both sides)

$(x + y)^2 = 5$; $(x - y)^2 = 3$

Solve the equations

$(x + y)^2 = x^2 + y^2 + 2xy = 5$

$(x - y)^2 = x^2 + y^2 - 2xy = 3$

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$2(x^2 + y^2) = 8$

$(x^2 + y^2) = \frac{8}{2} = 4$

$= x^2 + y^2 + 2xy = 5$

$= x^2 + y^2 - 2xy = 3$

(-) (-) (+) (-)

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$4xy = 2$

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$xy = \frac{2}{4} = \frac{1}{2}$

$8xy(x^2 + y^2) = 8 \times \frac{1}{2} \times 4$

$= 4 \times 4 = 16$

6. If $(a - \frac{1}{a}) = 6$. Find the value of $a^2 + \frac{1}{a^2}$

$\left(a - \frac{1}{a}\right) = 6$

$\left(a - \frac{1}{a}\right)^2 = 6^2$ (take square on both sides)

$a^2 + \left(\frac{1}{a}\right)^2 - 2(a)\left(\frac{1}{a}\right) = 36$

$a^2 + \frac{1}{a^2} - 2 = 36$

$a^2 + \frac{1}{a^2} = 36 + 2$

$a^2 + \frac{1}{a^2} = 38$

7. If $x - \frac{1}{x} = 2$, Find the value of $x^3 - \frac{1}{x^3}$

$(a - b)^3 = a^3 - b^3 - 3\,ab\,(a - b)$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\,(x)\left(\frac{1}{x}\right).\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$2^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$8 = x^3 - \frac{1}{x^3} - 3\,(2)$$

$$8 = x^3 - \frac{1}{x^3} - 6$$

$$8 + 6 = x^3 - \frac{1}{x^3}$$

$$14 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 14$$

8. If $x - \frac{1}{x} = 4$, Find the value of $x^4 + \frac{1}{x^4}$

$x - \frac{1}{x} = 4$ (take square on both sides)

$$\left(x - \frac{1}{x}\right)^2 = 4^2 \quad [(a - b)^2 = a^2 + b^2 - 2ab]$$

$$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 4^2$$

$$x^2 + \frac{1}{x^2} - 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 + 2$$

$$x^2 + \frac{1}{x^2} = 18$$

$\left(x^2 + \frac{1}{x^2}\right)^2 = (18)^2$ (take square on both sides)

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 324$$

$$x^4 + \frac{1}{x^4} + 2 = 324$$

$$x^4 + \frac{1}{x^4} = 324 - 2$$

$$x^4 + \frac{1}{x^4} = 322$$

## Assignment

**I. Add**

1) $45b + 25c + 18b + 40c$
2) $14d + 3d + 25e + 2e + e + d$
3) $15a - (4a + 3a - 5a)$
4) $5x + 3y - (2x - 5y)$
5) $(x + 2y + 3z) + (4x - y + z)$
6) $(2x + 5y) + (4x - 8z) + (15z - 6y) + (z - 2x)$
7) $(5x^2 - 3y^2 + z) + (-x^2 + 2y^2 - 4z)$
8) $(-2x + 3y - 3z) + (-6y - 5x + z)$
9) $(a - 3b + 4c) + (-7c - a + 4b)$
10) $(2x + 5y) + (4x - 8z) + (15z - 2y)$

**II. Subtract**

1) $38xy - 25xy$
2) Subtract $2a^2 - 3b^2$ from $3a^2 + 2b^2$
3) Subtract $-2y^2 + 3xy - 5$ from $3x^2 - 4xy + 7y^2 - 5$
4) Subtract $2a - 3b - c$ from $3a - 2b + 4c$
5) Subtract $3x - 4x^2 + 2y^2$ from $4y^2 - 2x + 8x^2$

## III. Simplify

1) $230a + 420b + 370c + 225a - 510c - 110b$

2) $15d - (4d + 3d - 5d)$

3) $2a - 3(a - (a - b))$

4) $48m^2 + 24m^2n + 12m^2 - 6m^2 - 12m^2n$

5) $8x + 3z - 6y - 5z + 4x + 8y$

6) $3x^2y - 2xy - 2x^2y - 3xy - 2x^2y + xy$

7) $10x + 14 - 7y^2 - 11a + 2x - 4 - 3y^2 - 4a + 8$

## IV.

1) $7pq^2 \text{ x } 5r$

2) $5yzx \text{ x } (-5ab)$

3) $(4x^2 + 3y^2) \text{ x } (-2z)$

4) $3ax \text{ -}9b$

5) $-7p \text{ x } 4q^2$

6) $2ab \text{ x } -7pq$

7) $p^2q^3 \text{ x } 3p^3q^2$

8) $(3b^2 - 2b)3b^2$

9 $5y \text{ x } 2y^3y^2$

10) $ab^{-1} \text{ x } ba^{-1}$

## V.

1) $\frac{10a}{2a}$

2) $-3ax \div -6x$

3) $15xy \div -5$

4) $-\frac{8ac}{2bc}$

5) $\frac{-5m \text{ x } -6n - 7p}{-28\,mn}$

6) $4a^8 \div 2a^3$

7) $-15a^8 \div 3a^5$

8) $\frac{8a^4}{12a^{-7}}$

9) $\frac{3p^2 \text{ x } 4p \text{ x } 5p^3 \text{ x } p}{6p^4 \text{ x } 10p}$

10) $\frac{25m^2n}{5m^3n^2}$